KINEMATICS IN 2D

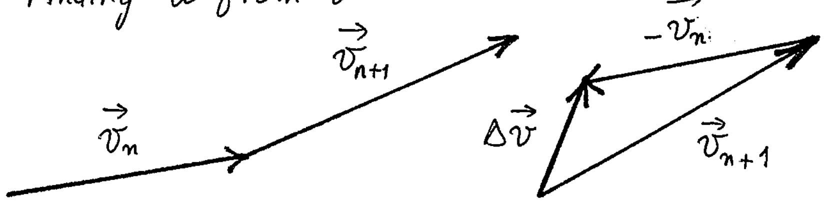
4.1 Acceleration

Average acceleration $\boxed{\vec{a}_{avg} = \frac{\Delta \vec{v}}{\Delta t}}$

$\vec{v}$ can change in 2 possible ways:

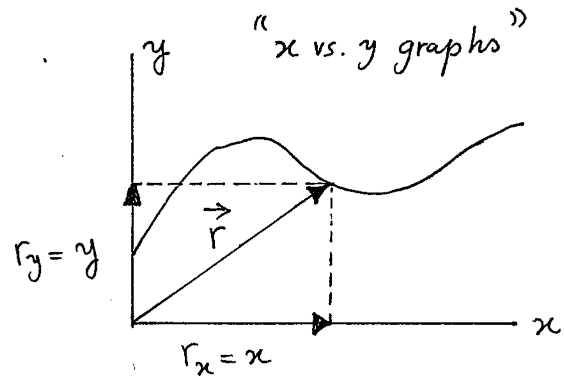
1) Its magnitude (speed) may change
2) Its direction may change.

Finding $\vec{a}$ from $\vec{v}$



$$\vec{a} = \frac{\Delta \vec{v}}{\Delta t}$$

4.2 Two Dimensional Kinematics



* Position vector $\boxed{\vec{r} = x\,\hat{i} + y\,\hat{j}}$

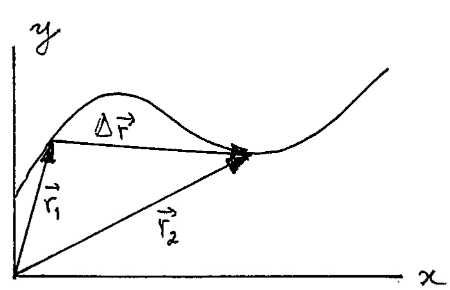
* Displacement vector

$$\Delta\vec{r} = \vec{r}_2 - \vec{r}_1$$

$$\Delta\vec{r} = (x_2\hat{i} + y_2\hat{j}) - (x_1\hat{i} + y_1\hat{j})$$

$$\Delta\vec{r} = \underbrace{(x_2 - x_1)}_{\Delta x}\hat{i} + \underbrace{(y_2 - y_1)}_{\Delta y}\hat{j}$$

$$\boxed{\Delta\vec{r} = \Delta x\,\hat{i} + \Delta y\,\hat{j}}$$



* Average velocity

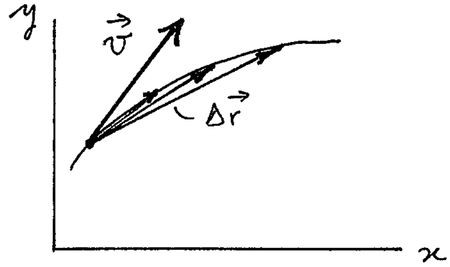
$$\boxed{\vec{v}_{avg} = \frac{\Delta\vec{r}}{\Delta t} = \frac{\Delta x}{\Delta t}\hat{i} + \frac{\Delta y}{\Delta t}\hat{j}}$$

* Instantenous velocity

$$\boxed{\vec{v} = \frac{d\vec{r}}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}} \rightarrow \vec{v} = v_x\hat{i} + v_y\hat{j}$$

$$v_x = \frac{dx}{dt} \quad \text{and} \quad v_y = \frac{dy}{dt}$$

The instantaneous velocity is tangent to the trajectory!



You can break up $\vec{v}$ to its x and y components given $|\vec{v}|$ and θ.

$$\boxed{\begin{cases} v_x = v\cos\theta \\ v_y = v\sin\theta \end{cases} \qquad v_x^2 + v_y^2 = v^2 \text{ and } \tan\theta = \frac{v_y}{v_x}}$$

* Do not confuse x-y graphs with x-t graphs.
* x vs y graphs picture the actual trajectory in space.
* Acceleration : $\vec{a} = \frac{d\vec{v}}{dt} = \underbrace{\frac{dv_x}{dt}}_{a_x}\hat{\imath} + \underbrace{\frac{dv_y}{dt}}_{a_y}\hat{\jmath}$

<u>Kinematics with constant acceleration</u>

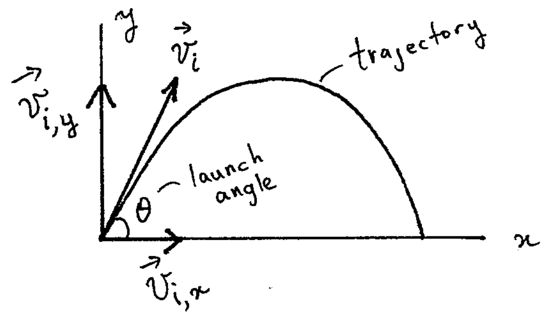
Once all the vectors $(\vec{r}, \vec{v}, \vec{a})$ decomposed into x and y components, equations for 1d-Kinematics apply to their components.

$$\begin{cases} x_f = x_i + v_{x,i}\,\Delta t + \frac{1}{2}a_x \Delta t^2 \;;\; v_{f,x} = v_{i,x} + a_x \Delta t \\ y_f = y_i + v_{y,i}\,\Delta t + \frac{1}{2}a_y \Delta t^2 \;;\; v_{f,y} = v_{i,y} + a_y \Delta t \end{cases}$$

4.3 Projectile Motion

This can be thought of as an extension of free fall motion.

We will neglect air resistance.



$$\vec{v}_i = \vec{v}_{i,x} + \vec{v}_{i,y} = v_{i,x}\,\hat{\imath} + v_{i,y}\,\hat{\jmath}$$

$$\boxed{\begin{cases} v_{ix} = v_i \cos\theta \\ v_{iy} = v_i \sin\theta \end{cases}}$$

The vertical (y) component of acceleration is $-g$.
The horizontal (x) component of acceleration is 0.

$$\boxed{\begin{aligned} a_x &= 0 \\ a_y &= -g \end{aligned}}$$

$a_x = 0$ ← motion w/ constant velocity in x-direction
$a_y = -g$ ← free fall in y-direction

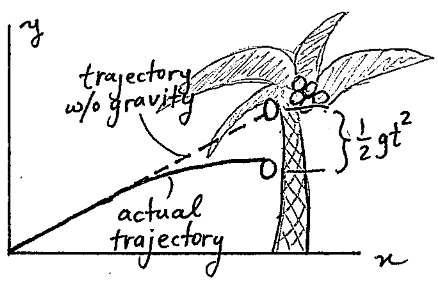
Therefore we can write kinematic equations of motion as the motion is made up from two <u>independent</u> components.

$$\boxed{\begin{aligned} x_f &= x_i + v_{i,x}\,\Delta t \;;\quad v_{f,x} = v_{i,x} = \text{constant} \\ y_f &= y_i + v_{i,y}\,\Delta t - \tfrac{1}{2} g (\Delta t)^2 \;;\quad v_{f,y} = v_{i,y} - g\,\Delta t \end{aligned}}$$

Consider the following question:

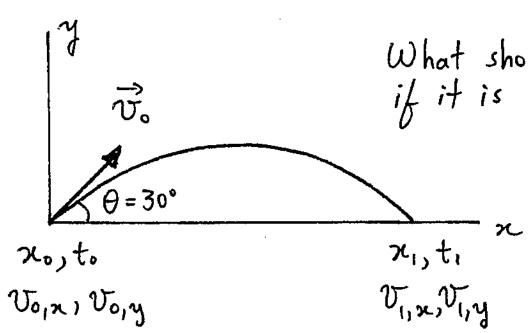
"A ball is thrown exactly horizontally at height h above the horizontal ground. At the same time, a second ball is simly dropped from the same height. Which ball hits the ground first?"

Also consider the coconut tree problem.



The actual trajectory always "falls" $\frac{1}{2}gt^2$ below the the straight line.

PROB



What should be the speed of the particle if it is to land 100 m ahead?

$$\begin{cases} x_1 = x_0 + v_{0,x}(t_1 - t_0) \\ y_1 = y_0 + v_{0,y}\,\Delta t - \frac{1}{2} g\,\Delta t^2 \end{cases}$$

$$\begin{cases} x_1 = v_0 \cos\theta\, t_1 \quad \text{.........(1)} \\ 0 = v_0 \sin\theta\, t_1 - \frac{1}{2} g t_1^2 \quad \text{....(2)} \end{cases}$$

From the second equation (2)

$$\left(v_0 \sin\theta - \frac{1}{2} g t_1\right) t_1 = 0 \rightarrow t_1 = \begin{cases} 0 \\ \dfrac{2 v_0 \sin\theta}{g} \end{cases}$$

Substitute t_1 in first equation (1)

$$x_1 = \frac{2 v_0^2 \sin\theta \cos\theta}{g}$$

$$x_1 = \frac{v_0^2 \sin 2\theta}{g}$$

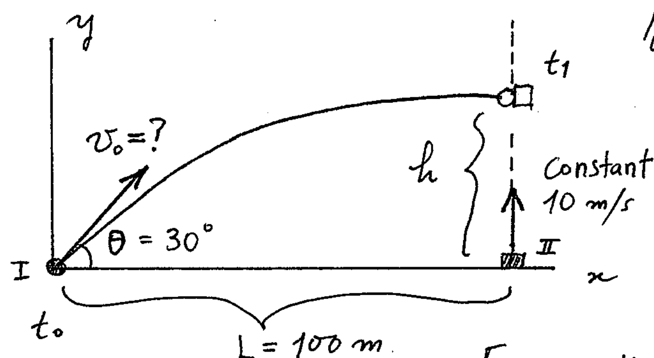
Notice two things
1) Range is maximum when $\theta = 45°$
2) Range for θ and $(90° - \theta)$ are the same

$$\sin[\underbrace{2(90-\theta)}_{\text{launch angle}}] = \sin(180 - 2\theta) = \sin 2\underbrace{\theta}_{\text{launch angle}}$$

$$100\,m = \frac{v_0^2 \sin 60°}{9.80}$$

$$v_0 = 33.6\ \frac{m}{s}$$

PROB



If two particles are to collide, what should v_0 be?

$$L = v_{0,x}^{I} (t_1 - t_0)$$

$$L = v_0^{I} \cos\theta\, t_1$$

$$t_1 = \frac{L}{v_0^{I} \cos\theta}$$

For particles to collide in the air, they both have to travel same distance in y-direction in time t_1.

$$h = \cancel{y_0^{I}}^{\,0} + v_0^{I} \sin\theta\, t_1 - \frac{1}{2} g t_1^2 \quad \text{---- (1)}$$

The verticle distance travelled by particle II in time t_1,

$$h = v_{0,y}^{II} t_1 \quad \text{---- (2)}$$

Equate equations (1) and (2)

$$v_0^{I} \sin\theta\, \cancel{t_1} - \frac{1}{2} g t_1^{\cancel{2}} = v_0^{II} \cancel{t_1} \quad \left(\text{Note that } v_{0,y}^{II} = v_0^{II}\right)$$

$$v_0^{I} \sin\theta = v_0^{II} + \frac{1}{2} g t_1$$

$$v_0^{I}\sin\theta - \frac{1}{2} g t_1 - v_0^{II} = 0$$

Substitute $t_1 = L / v_0^{I}\cos\theta$ to get

$$v_0^{I}\sin\theta - \frac{1}{2} g \frac{L}{v_0^{I}\cos\theta} - v_0^{II} = 0$$

$$\times 2 \Big/ \quad (v_0^{I})^2\sin\theta\cos\theta - \frac{1}{2} gL - v_0^{II} v_0^{I}\cos\theta = 0$$

Remember that $\theta = 30°$, $L = 100\ m$, $v_0^{II} = 10\ m/s$

$$0.866 (v_0^{I})^2 - 17.3\, v_0^{I} - 980 = 0$$

$$v_0^{I} = \begin{cases} 45\ m/s\ \checkmark \\ -25\ m/s\ \times \end{cases}$$

this is speed
hence <u>must be</u> positive.