CHAPTER 5 + 6

* What is force? Kinds of forces

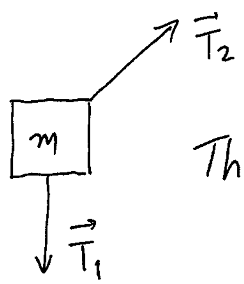
* $\underline{\text{Mass}}$ (m) : Your car is much harder to push than your bicycle. The tendency of an object to resist a change in its velocity is called inertia.

  Mass is an "intrinsic" property of an object.

* Newton's second law : The acceleration is determined by the net force F_{net}.

$$\vec{a} = \frac{\vec{F}_{net}}{m} \quad \text{where} \quad \vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \cdots = \sum_{i=1}^{N} \vec{F}_i$$

  * The acceleration is in the same direction as the net force.



$$\vec{a} = \frac{\vec{F}_{net}}{m} = \frac{\vec{T}_1 + \vec{T}_2}{m}$$

The amplitude of acceleration is NOT $\frac{T_1 + T_2}{m}$.

  * One Newton (N) is the force that causes 1 kg of mass to accelerate at $1\ m/s^2$.

$$1\ N = 1\ kgm/s^2$$

* Newton's First Law : Object at rest vs. (Object in motion with constant velocity)

  Greek's question : What causes objects to move.

  Newton's question : What causes an object to change its velocity.

  * Inertial reference frames : Ball sitting in a plane.

* Free Body Diagrams

CHAPTER 5

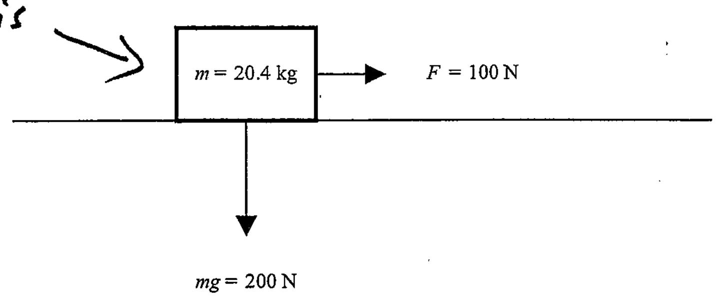
Vector Forces/Free Body Diagrams I

Newton's Laws:

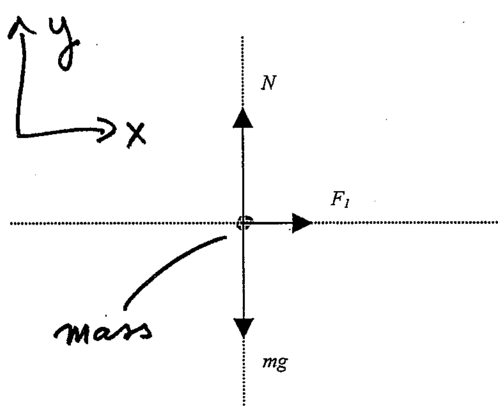
- The Law of Inertia
- $F = ma$
- Forces occur in pairs

Consider a system consisting of a block (mass = 20.4 kg) on a smooth, flat table pulled to the right by a force of 100 Newtons.

Start w/ this



The *Free Body Diagram* (FBD) for this system:



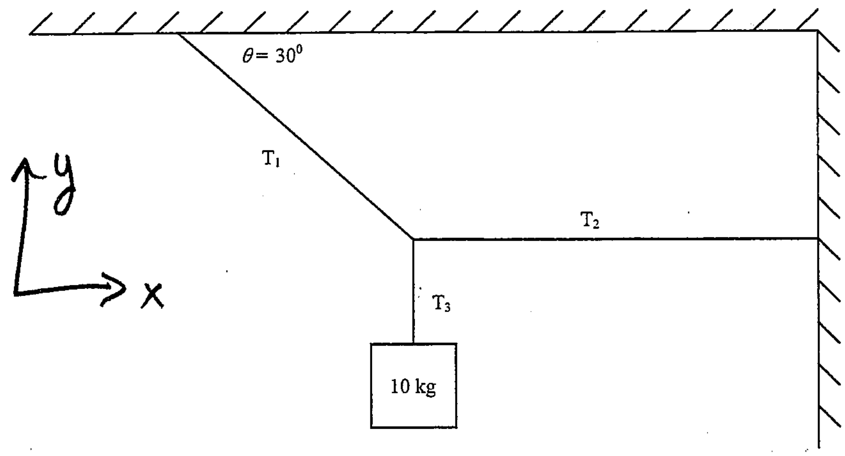
Note:

- the forces in the y direction are balanced (i.e., $mg = N$) while the forces in the x direction are not.
- the forces in this example lie along the x and y-axes.

Let's write Newton's second law for the sum of the forces along each axis.

$$\sum F_x = +F_1 = ma$$
$$\sum F_y = +N - mg = 0 \therefore N = mg$$

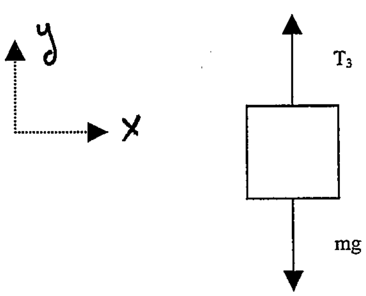
Example 1.



- In this particular case, the system consists of the block and three light cords anchoring it to a ceiling and wall. The mass of the block is known.
- We seek the tension in each of the connecting cords.
- In this example all of the forces are balanced and the system is said to be in *equilibrium* ($\Sigma F = 0$), i.e., no accelerations are present.
- In order to perform a vector analysis of this system we will need to produce FBD's that will yield sufficient equations to solve for all the unknown's in the system, T_1, T_2, T_3 (three unknowns require three equations).

Let's look first at the hanging mass:

FBD of the mass

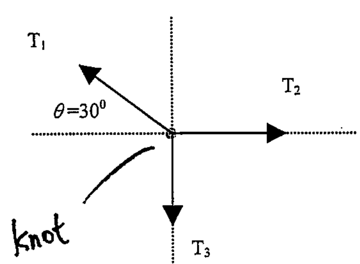


$$\left[\begin{array}{l}\sum F_x = 0 \\ \sum F_y = T_3 - mg = 0\end{array}\right. \qquad \textbf{(1)}$$
$$\therefore T_3 = mg = 98N$$

This FBD yields one useful equation ($T_3 = 98N$).

Let's try a FBD of the "knot" where the cords are joined. This will yield two of the three equations we need.

FBD of the "knot"

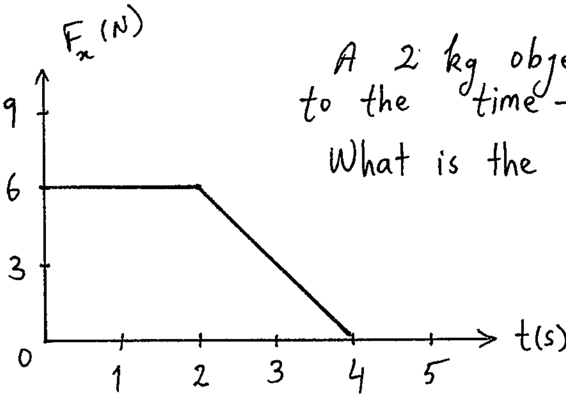


$$\left[\sum F_x = T_2 - T_1\left(\cos 30^0\right) = 0\right. \qquad \textbf{(2)}$$
$$\therefore T_2 = T_1\left(\cos 30^0\right)$$

$$\left[\sum F_y = T_1\left(\sin 30^0\right) - T_3 = 0\right.$$
$$\therefore T_1\left(\sin 30^0\right) = T_3 = 98N \qquad \textbf{(3)}$$
$$\therefore T_1 = 196N$$

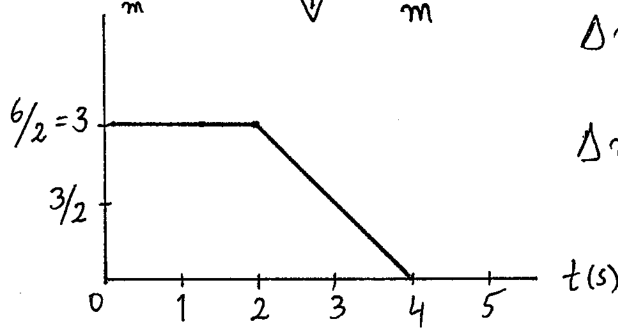
Equation (2) now yields $T_2 = 169.7N$ and T_1 = **196*N***, T_2 = **169.7*N***, T_3 = **98*N***

PROB 28



A 2 kg object initially at rest is subjected to the time-varying force on the left. What is the object's velocity at $t = 4\,s$?

$\frac{F_x}{m} = a_x$ $\downarrow$ $\frac{F_x}{m} = a_x$



$$\Delta v_x = \int_{t_i}^{t_f} a\, dt$$

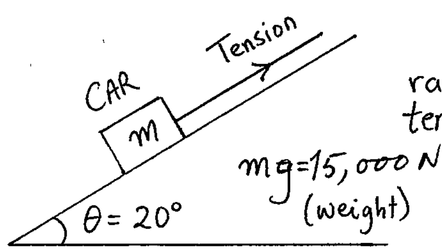
$$\Delta v_x = \begin{cases} \text{Area under } a_x \text{ vs. } t \text{ graph} \\ \text{between } t_i \text{ and } t_f \end{cases}$$

$$\Delta v_x = \frac{1}{2}(2+4)\cdot 3$$

$$\Delta v_x = 9\ \frac{m}{s}$$

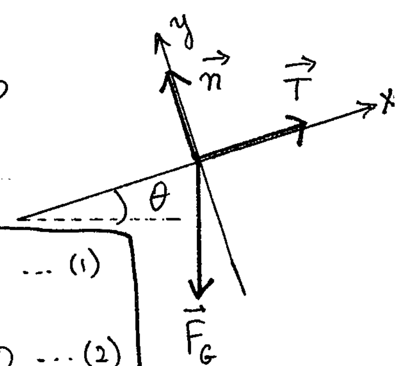
Initially at rest $\rightarrow$ $v_f = 9\ \frac{m}{s}$.

PROB



A car is being towed up on a 20° ramp at constant velocity. What is the tension on the tow rope.

* Draw a FBD (Free Body Diagram) w/ coordinate axes.



Since $\vec{a} = \vec{0}$ then $a_x = 0$ and $a_y = 0$

$\boxed{a_x = \frac{\sum F_x}{m} = 0}$ and $\boxed{a_y = \frac{\sum F_y}{m} = 0.}$

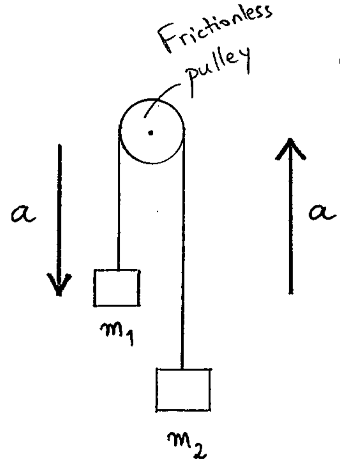
$$\boxed{\begin{aligned} (F_{net})_x &= \sum F_x = T - F_G \sin\theta = 0 \quad \dots (1) \\ (F_{net})_y &= \sum F_y = n - F_G \cos\theta = 0 \quad \dots (2) \end{aligned}}$$

From Eq. (1) $T = F_G \sin\theta$

$T = mg \sin\theta = 15{,}000\,N \cdot \sin 20°$

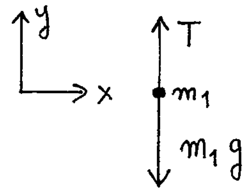
$T = 5100\ N$

PROB Atwood's Machine



If $m_1 > m_2$, find the tension in the connecting cord and the acceleration of the system.

FBD #1 :

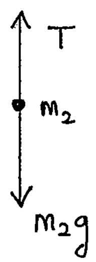


$$\sum F_x = 0$$

$$\sum F_y = T - m_1 g = -m_1 a$$

$$\boxed{T = m_1 g - m_1 a} \quad \text{----- (1)}$$

FBD #2 :



$$\sum F_x = 0$$

$$\sum F_y = T - m_2 g$$

$$= m_2 a \rightarrow \boxed{T = m_2 g + m_2 a} \quad \text{---- (2)}$$

Combining Eqs. (1) and (2)

$$m_1 g - m_1 a = m_2 g + m_2 a$$

$$(m_1 + m_2) a = (m_1 - m_2) g$$

$$\boxed{a = \frac{m_1 - m_2}{m_1 + m_2} g} \rightarrow \text{acceleration}$$

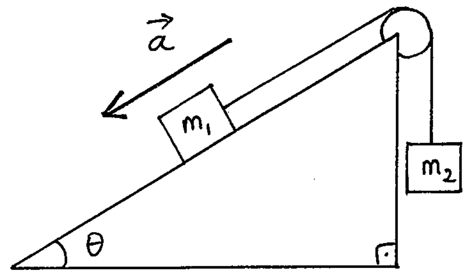
Substituting a in Eq (1) gives Tension,

$$T = m_1 (g - a)$$

$$T = m_1 \left(1 - \frac{m_1 - m_2}{m_1 + m_2} \right) g$$

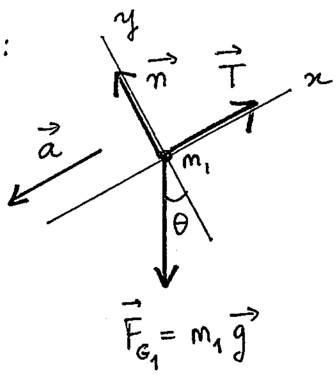
$$\boxed{T = m_1 \frac{2 m_2}{m_1 + m_2} g}$$

PROB



If $m_1 > m_2$ AND given the direction of the acceleration, find the magnitude of the acceleration.

FBD #1 :

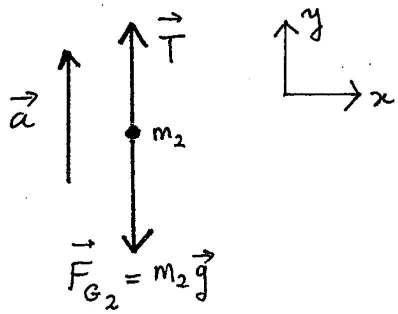


$$\sum F_x = -m_1 g \sin\theta + T = m_1(-a)$$

$$\boxed{T = m_1 g \sin\theta - m_1 a} \quad \text{----(1)}$$

$$\sum F_y = n - m_1 g \cos\theta = 0$$

FBD #2 :



$$\sum F_x = 0$$

$$\sum F_y = T - m_2 g = m_2 a$$

$$\boxed{T = m_2 g + m_2 a} \quad \text{...(2)}$$

Setting Eqns. (1) and (2) equal since T is same throughout

$$m_1 g \sin\theta - m_1 a = m_2 g + m_2 a$$

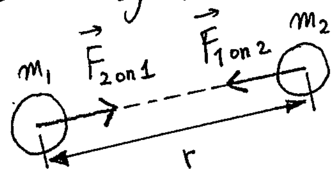
$$(m_1 + m_2) a = m_1 g \sin\theta - m_2 g$$

$$\boxed{a = \frac{m_1 \sin\theta - m_2}{m_1 + m_2} g}$$

6.3 Mass, Weight and Gravity

* **Mass:**
  * Measured using a pan balance.
  * Independent of gravity. (Consider measurement on different planets!)
  * Intrinsic property of the object. It tells us <u>nothing</u> about where the object is, what are its velocity, acceleration, what forces are acting on it etc.

* **Gravity:** Gravity is an attractive, long range <u>force</u> between two objects.



$$F_{2on1} = F_{1on2} = \frac{G m_1 m_2}{r^2}$$

(Newton's Law of gravity)

For a free falling object $\vec{F}_{net} = \vec{F}_G$

$$F_G = \frac{GmM}{(r+R)^2} \simeq \frac{GmM}{R^2} \quad \text{since } \underset{\substack{\uparrow \\ \text{height} \\ \text{from ground}}}{r} \ll \underset{\substack{\uparrow \\ \text{radius of} \\ \text{earth.}}}{R}.$$

$$\frac{GmM}{R^2} = mg \rightarrow g = \frac{GM}{R^2} \rightarrow \text{towards the center of earth.}$$

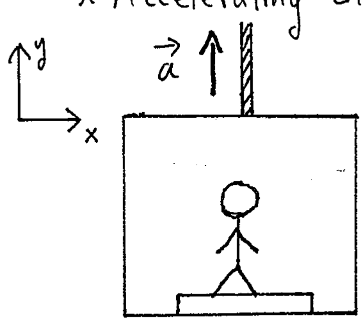
$$\begin{cases} R = 6.37 \times 10^6 \, m \\ M = 5.98 \times 10^{24} \, kg \end{cases}$$

* **Weight:** Weight is result of a measurement done by a spring scale. Weight is a force. (F_{sp})

$$w = F_{sp} = F_G = mg$$

* Mass and weight are not the same thing.

* Accelerating Elevator:



A person is standing on a scale in an elevator which is accelerating.

$w =$ scale reading $F_{sp} = mg + ma_y$; $a_y = a$

$w = mg\left(1 + \frac{a_y}{g}\right)$, when $a = 0$ we recover $w = mg$.

6.4 Friction

Static Friction: It keeps object from slipping.
It is in the opposite direction in which the object would move if there was no friction.

$$\boxed{\begin{array}{l} \vec{f}_s = -\vec{F}_{push} \\ f_s = F_{push} \end{array}}$$

* f_s depends on how hard the object is pushed/pulled.
* f_s has a maximum possible size : $f_{s,max}$.
  1) If $f_s < f_{s,max}$, the object remains at rest.
  2) If $f_s = f_{s,max}$, the object slips.
  3) $f_s > f_{s,max}$ is not possible.
* Again, experiments have shown that $\boxed{f_{s,max} = \mu_s n}$
* μ_s is called coefficient of static friction and depends on the materials of which the object and the surface are made of, their roughness, cleanliness etc.

Kinetic Friction: Once the the object starts to slide, f_s is replaced by f_k.

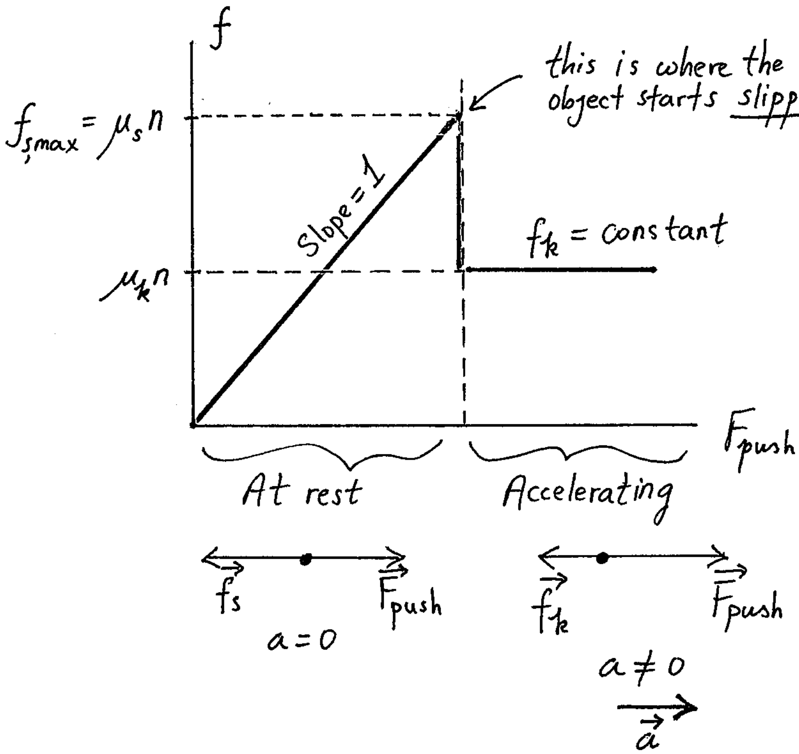
$\boxed{f_k < f_{s,max}}$ → why it is harder get the object moving than to keep it moving.

* Direction of f_k is alway opposite to the direction in which the object is sliding (moving).

$\boxed{f_k = \mu_k n}$; μ_k : Kinetic friction coefficient.

$\boxed{\mu_k < \mu_s}$ since $f_k < f_{s,max}$

Rolling Friction : $f_r = \mu_r n$ and $\mu_r \ll \mu_k$.



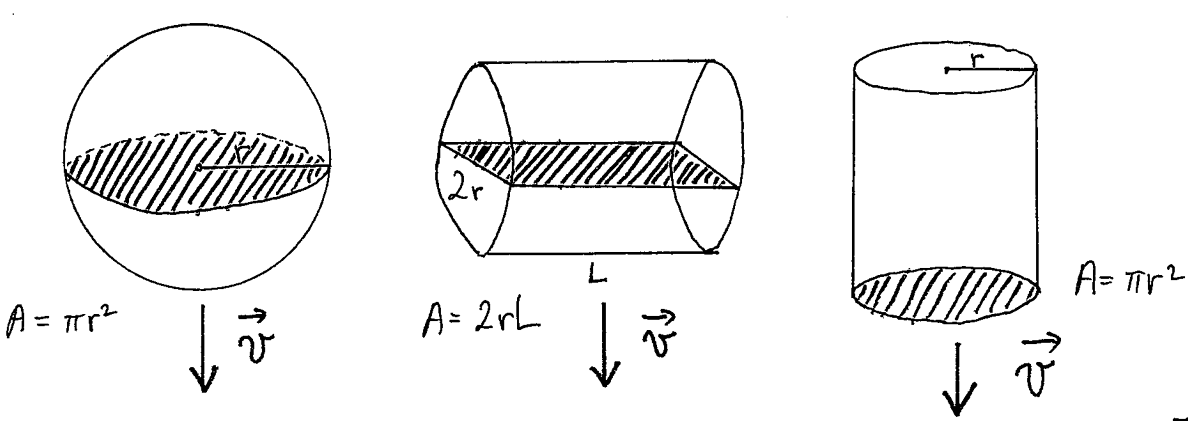
6.5 Drag

* Drag is in opposite direction to $\vec{v}$.
* It increases in magnitude as object's speed increases.

Our model for Drag force $\rightarrow$ $\vec{D} \approx \frac{1}{4} A v^2$ (direction opposite to motion)

$\frac{1}{4}$: ~ Air density

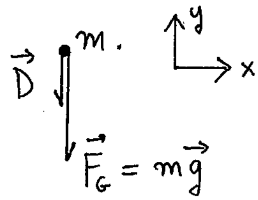
A: Cross sectional area perpendicular to $\vec{v}$



A ball tossed up:

1) The drag force points down as ball rises. F_{net} on ball increases.

FBD:



$\sum F_x = 0$

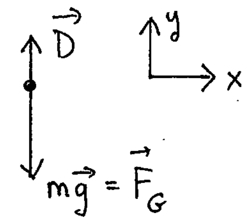
$\sum F_y = -D - mg = ma_\uparrow$

$$\boxed{a_\uparrow = -\left(g + \frac{D}{m}\right)}$$

2) At top, $\vec{D} = \vec{0}$ and $a = -g$.

3) As the ball falls, $\vec{D}$ points up. F_{net} on ball decreases.

FBD:



$\sum F_x = 0$

$\sum F_y = D - mg = ma_\downarrow$

$$\boxed{a_\downarrow = -\left(g - \frac{D}{m}\right)}$$

* Notice that D is same for two objects of equal size. However, if the masses are different, they accelerate differently.

<u>Example</u>: * Throwing a golf ball vs. Throwing a ping pong ball.
* A bowling ball vs. a feather.

Terminal Speed:

Since D increases with speed, when speed reaches a value $|v_{term}|$ for which D matches $mg = F_G$, the acceleration a becomes zero since $\vec{F}_{net} = \vec{0}$. The object keeps falling at constant speed $|v_{term}|$.

$$\boxed{\frac{1}{4} A v_{term}^2 \approx mg} \rightarrow \boxed{v_{term} \approx \sqrt{\frac{4mg}{A}}}$$

* Note that the factor $\frac{1}{4}$ in D has units of $\frac{kg}{m^3}$. (Density)