CHAPTER 7

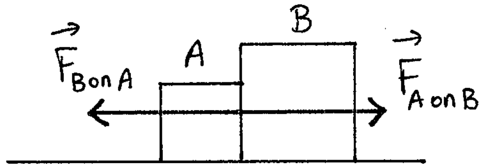
What we have done: Single particles responding to external forces.

What we will do: *Interacting objects*

Newton's 2nd law is not sufficient to describe dynamics of interacting objects alone.

Consider hammer+nail example: Any time an object A pushes or pulls and object B, the object B pushes or pulls back on the object A.

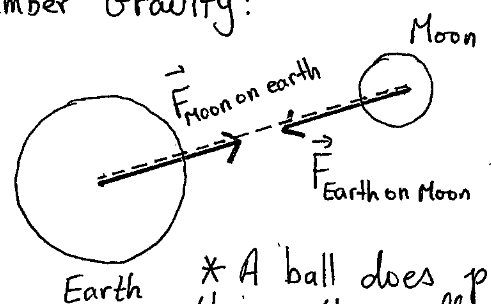
An <u>interaction</u> is the mutual influence of two objects together.



A and B constitute an action/reaction pair.

<u>Important:</u> Action/reaction pair exists as a <u>pair</u> or <u>not at all.</u>

Remember Gravity:



$$F_G = \frac{GMm}{R^2}$$

$$F_G = M\left(\frac{Gm}{R^2}\right) = m\left(\frac{GM}{R^2}\right)$$

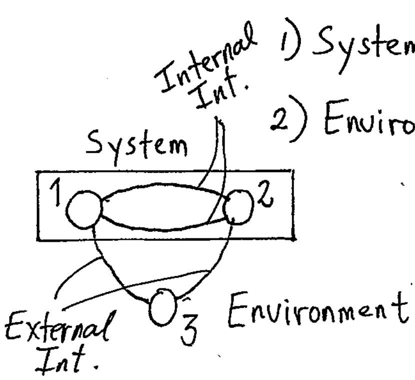
* A ball does pull upward on the earth as the earth pulls downward on the ball.

⇒ We want to extend the particle model to situations in which 2 or more objects <u>interact</u>.

⇒ Divide the interacting objects into two groups:

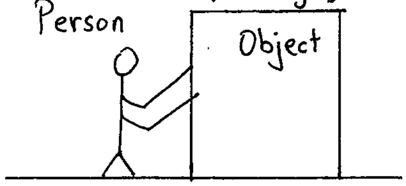
1) System: Objects whose motion we <u>want</u> to analyze.

2) Environment: Objects which <u>interact w/ the system</u> but resulting <u>motions of which are irrelevant</u>.

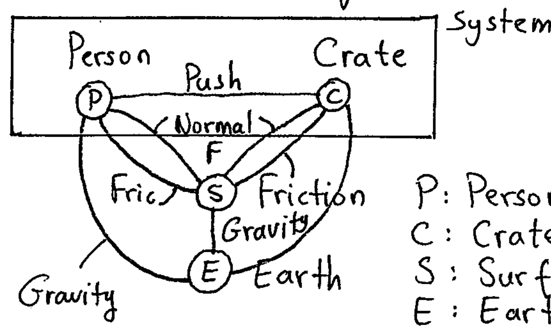


Example

Pushing a crate.
(sliding!) →

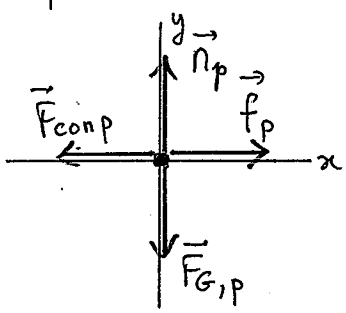


The interaction diagram

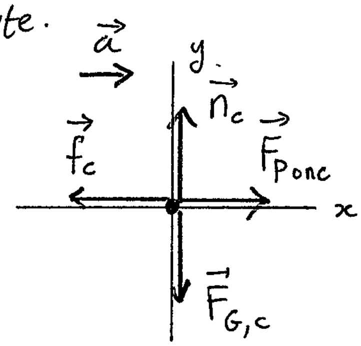


P: Person
C: Crate
S: Surface
E: Earth

Free Body Diagrams for the person and the crate.



Person



Crate

Be careful about the force of friction!

$\vec{f}_p$ is $\vec{f}_{S\,on\,P}$

Also note : The forces of action/reaction pair never act on the same object.

Propulsion: * Walking on a floor. Also consider walking on a frictionless floor. (Static friction)

* Car driving along a road. (Static friction)
* Thrust force of a rocket.

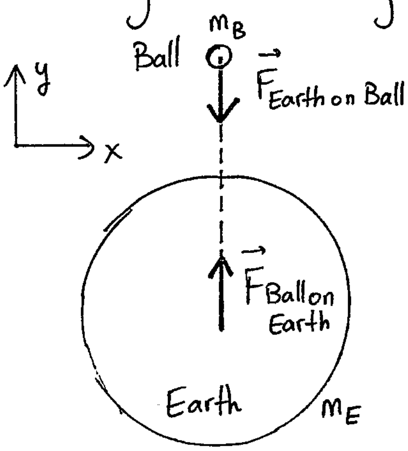
7.3 Newton's Third Law

Every force occurs as one member of an action/reaction pair of forces.

* The two members of an action/reaction pair act on two different objects.
* $\vec{F}_{A\,on\,B} = -\vec{F}_{B\,on\,A}$

* Newton's 3rd law extends & completes our concept of force. We can now recognize force as an <u>interaction</u> rather than ~~rather~~ than a "thing" with an independent <u>existence</u> of its own.

Revisiting Free Falling Object:



$$\vec{F}_{\text{Earth on Ball}} = \vec{F}_{G,B} = -\vec{F}_{\text{Ball on Earth}}$$

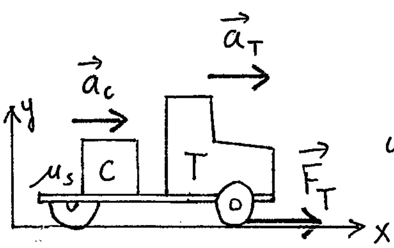
$$\vec{a}_B = \frac{\vec{F}_{G,B}}{m_B} = -g\,\hat{j}$$

$$\vec{a}_E = \frac{\vec{F}_{\text{Ball on Earth}}}{m_E} = \frac{-\vec{F}_{G,B}}{m_E} = \frac{-(-m_B g\,\hat{j})}{m_E}$$

$$\vec{a}_E = \left(\frac{m_B}{m_E}\right) g\,\hat{j}$$

$$|\vec{a}_E| \cong \frac{1\ \text{kg}}{6\times 10^{24}\ \text{kg}}\, g \approx 2\times 10^{-24}\ \text{m/s}^2$$ → It would take ~500,000 times age of the universe for earth to reach 1 mph!

<u>PROB</u>

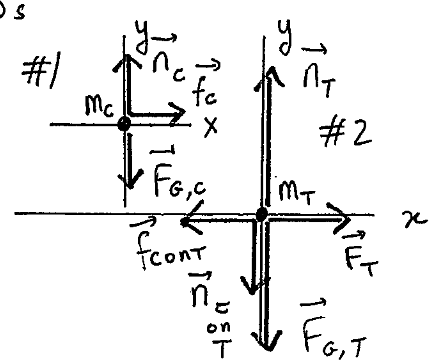


What is the maximum magnitude $\vec{F}_T$ can have without the crate sliding?

$$\left.\begin{matrix} m_T = 2000\ \text{kg} & m_C = 200\ \text{kg} \\ \mu_s = 0.80 & \mu_k = 0.30 \end{matrix}\right\} F_T = ?$$

We know that $|\vec{a}_c| = |\vec{a}_T| = a_x$

Draw FBDs



Newton's 2nd law : <u>FBD #1</u>

$$\begin{cases} \sum (F_{\text{net on }c})_x = f_{T\,on\,c} = m_c a_x \\ \sum (F_{\text{net on }c})_y = n_{T\,on\,c} - m_c g = m_c \cancel{a_y} \ \ (=0) \end{cases}$$

$$n_{T\,on\,C} = m_c g$$

$$f_{T\,on\,C} = n_{T\,on\,C}\,\mu_s = m_c g \mu_s$$

$$a_x = \frac{f_{T\,on\,c}}{m_c} = g\mu_s$$

Therefore crate's max acceleration w/o slipping

$$\boxed{a_x = \mu_s g}$$

FBD #II (Truck): $\sum (F_{\text{net on } T})_x = F_T - f_{cT} = m_T a_x$

$\boxed{F_{AB} \text{ means } F_{A \text{ exerting on } B}}$

$$\sum (F_{\text{net on } T})_y = n_T - n_{cT} - m_T g = 0$$

From x-equation: $\boxed{F_T = m_T a_x + f_{cT}}$

Newton's 3rd law $\rightarrow$ $f_{cT} = f_{Tc} = m_c g \mu_s$, $a_x = g\mu_s$

$$\boxed{F_T = m_T g \mu_s + m_c g \mu_s}$$

$$F_T = (m_T + m_c)\, g\, \mu_s = (2200\ \text{kg})(9.80\ \text{m/s}^2)\, 0.80$$

$$= 17{,}000\ \text{N}$$

$\Rightarrow$ Note that we only needed FBD #1 to get a_x, but needed FBD #2 to get F_T.

7.4 Ropes and Pulleys

In single particle dynamics, we defined tension as the force exerted on the object by a rope/string.

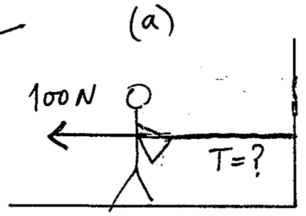
What do we mean by "tension" when we talk about objects interacting via a string attached to both of them?

Molecular spring model for tension in the rope (Ch. 05).

Important: Tension pulls <u>equally</u> in both directions!

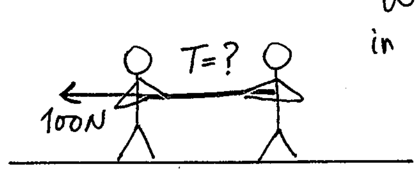
Example

(a)



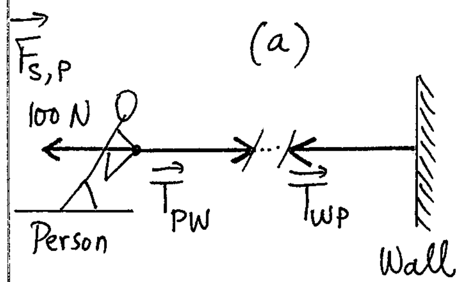
A person pulling a rope w/ 100 N force attached to a wall.

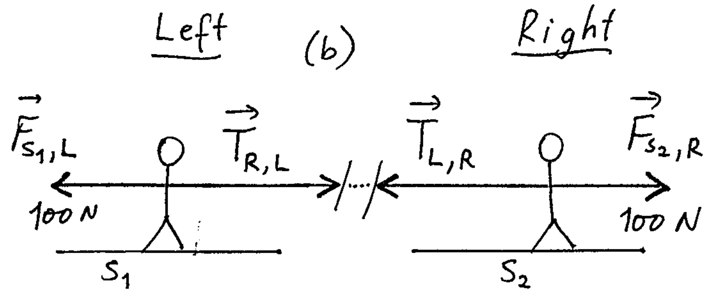
(b)



Two people in tug-of-war. Each pulling w/ 100 N force

What are tensions in ropes in (a) and (b)?



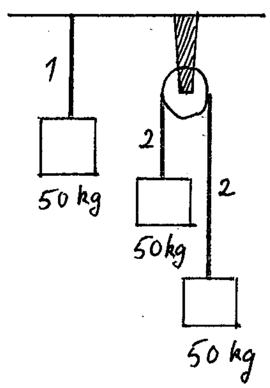


(a) $\begin{cases} T_{PW} = F_{SP} = 100\text{ N} \rightarrow \text{Newton's 2nd Law} \\ T_{PW} = T_{WP} = 100\text{ N} \rightarrow \text{Newton's 3rd Law} \end{cases}$

For the tug-of-war $\rightarrow$

(b) $\begin{cases} \left.\begin{aligned} F_{S_1,L} &= T_{R,L} = 100\text{ N} \\ F_{S_2,R} &= T_{L,R} = 100\text{ N} \end{aligned}\right\} \text{Newton's 2nd Law} \\ T_{R,L} = T_{L,R} = 100\text{ N} \rightarrow \text{Newton's 3rd Law} \end{cases}$

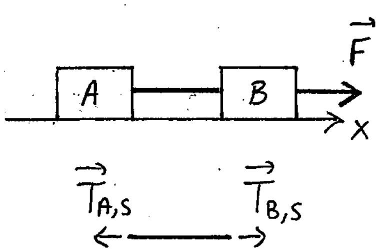
Example



$T_1 = T_2$. (Same as example above)

The examples above illustrated the "static" case.

What happens if the the system is acted by a force $\vec{F}$?



$(F_{net})_x = T_{BS} - T_{AS} = m_s a_x \rightarrow$ Net F on the string!

$T_{BS} > T_{AS}$

* Tension at the "front" of the string is higher than the tension at the back!

* When $a_x = 0$, we recover "static" condition.

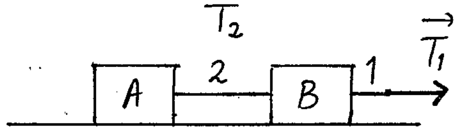
If we assume that mass of the string is negligible, i.e. $m_s = 0$,

$$T_{BS} - T_{AS} = m_s a_x = 0$$

$$\boxed{T_{BS} = T_{AS}}$$

* If objects A and B interact through a "massless" string, we can omit the string and treat F_{AB} and F_{BA} as an action/reaction pair.

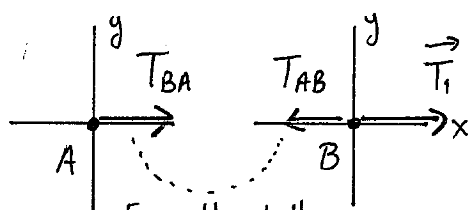
PROB



$m_B > m_A$

A and B are connected through a massless string.

Is the tension in string 2 equal, less than, or greater than T_1?



Even though the system has F_{net} action on it, we treat T_{BA} and T_{AB} as an action/reaction pair.

Newton's 3rd Law $\rightarrow$ $T_{BA} = T_{AB} = T_2$

Newton's 2nd Law

$$T_1 - T_{AB} = m_B a_{Ax}$$

$$T_1 - T_2 = m_B a_{Ax} > 0$$

$$\boxed{T_1 > T_2}$$

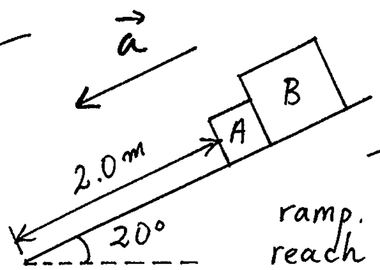
Pulleys: If $\rightarrow$ 1) the string and the pulley are massless, AND

2) there is no friction where the pulley turns on its axle

THAN...

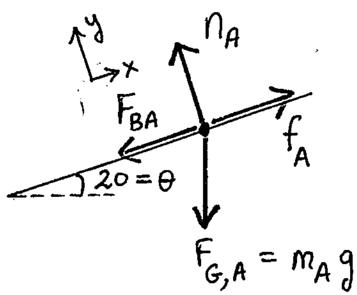
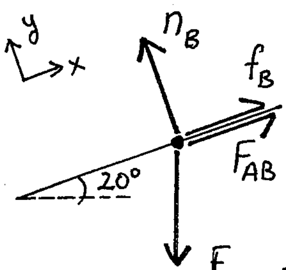
the tension remains constant as the string passes over a massless, frictionless pulley.

PROB 29



$$\begin{cases} m_A = 5.0 \text{ kg} & \mu_A = 0.20 \\ m_B = 10 \text{ kg} & \mu_B = 0.15 \end{cases}$$

Two packages start sliding down the ramp. How long does it take for them to reach the bottom?

FBD for A	FBD for B
	

FBD for A:

$$\sum F_x = f_A - F_{BA} - m_A g \sin\theta = m_A a_x$$

$$\sum F_y = n_A - m_A g \cos\theta = 0$$

$$\boxed{n_A = m_A g \cos\theta}$$

$$F_{BA} = f_A - m_A g \sin\theta - m_A a_x$$

$$F_{BA} = \mu_A m_A g \cos\theta - m_A g \sin\theta - m_A a_x$$

$$\boxed{F_{BA} = (\mu_A \cos\theta - \sin\theta) m_A g - m_A a_x}$$

FBD for B:

$$\sum F_x = F_{AB} + f_B - m_B g \sin\theta = m_B a_x$$

$$\sum F_y = n_B - m_B g \cos\theta = 0$$

$$\boxed{n_B = m_B g \cos\theta}$$

$$F_{AB} = m_B g \sin\theta - f_B + m_B a_x$$

$$\boxed{F_{AB} = m_B g(\sin\theta - \mu_B \cos\theta) + m_B a_x}$$

From Newton's 3rd Law $F_{BA} = F_{AB}$

$$(m_A + m_B) a_x = (\mu_A m_A + \mu_B m_B) g \cos\theta - (m_A + m_B) g \sin\theta$$

$$(15 \text{ kg}) a_x = (1.0 \text{ kg} + 1.5 \text{ kg})\, 9.2 \tfrac{m}{s^2} - 15 \text{ kg} (9.8 \tfrac{m}{s^2})\, 0.34$$

$$\boxed{a_x = -1.8 \tfrac{m}{s^2}}$$

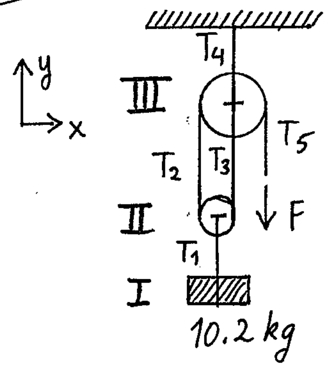
The rest of the problem is kinematical,

$$\boxed{\Delta x = v_0 \Delta t + \frac{1}{2} a_x (\Delta t)^2}$$

$$-2.0\,\text{m} = 0\,\text{m} + \frac{1}{2}\left(-1.8\,\frac{\text{m}}{\text{s}^2}\right)(\Delta t)^2$$

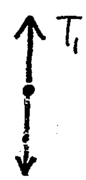
$$\boxed{\Delta t = 1.5\,\text{s}}$$

PROB 37



If the system on the left is static, find $T_{1,2,3,4,5}$ and F. (Both pulleys are frictionless and massless.)

FBD I

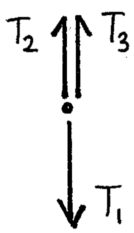


$$\sum F_x = 0$$

$$\sum F_y = T_1 - F_G = 0 \Rightarrow \boxed{T_1 = F_G = 100\,N}$$

$$F_G = mg = 100\,N$$

FBD II



$$\sum F_x = 0$$

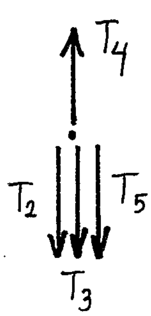
$$\sum F_y = T_2 + T_3 - T_1 = 0$$

$$T_1 = T_2 + T_3 = 100\,N$$

Pulleys are frictionless : $T_2 = T_3 = T_5 = F$

$$\boxed{T_2 = T_3 = 50\,N = T_5 = F}$$

FBD III

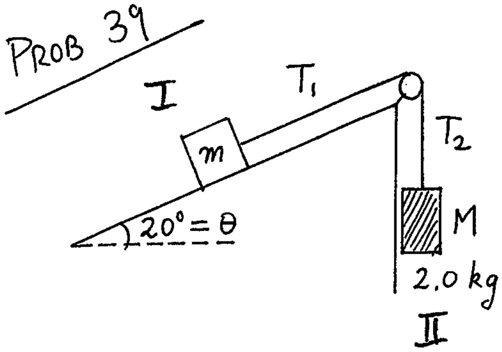


$$\sum F_x = 0$$

$$\sum F_y = T_4 - T_2 - T_3 - T_5 = 0$$

$$\boxed{T_4 = T_2 + T_3 + T_5 = 150\,N}$$

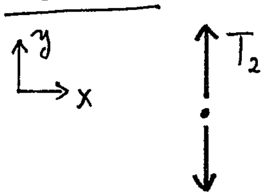
PROB 39



$\mu_s = 0.80$ and $\mu_k = 0.50$

a) What is the minimum mass m that will stick and not slip?

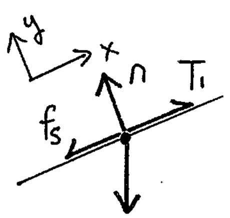
FBD II



$F_{G,M} = Mg = 19.6\ N$

$$\sum F_x = 0$$
$$\sum F_y = T_2 - Mg = 0$$
$$\boxed{T_2 = 19.6\ N}$$

FBD I



$F_{G,m} = mg$

$$\sum F_x = T_1 - f_s - mg\sin\theta = 0$$
$$\sum F_y = n - mg\cos\theta = 0$$
$$\boxed{n = mg\cos\theta}$$
$$T_1 = f_s + mg\sin\theta$$
$$\boxed{T_1 = (\mu_s \cos\theta + \sin\theta)\, mg}$$

From Newton's 3rd Law $\rightarrow$ $T_1 = T_2$ for a <u>frictionless</u> pulley.

$$mg\,(\mu_s \cos\theta + \sin\theta) = Mg = 19.6\ N$$
$$m\,(0.75 + 0.34) = 2\ kg$$
$$\boxed{m = 1.8\ kg}$$

b) If this minimum mass is nudged ever so slightly, it will start being pulled up the incline. What acceleration will it have?

From FBD I

$$\sum F_x = T_1 - f_k - mg\sin\theta = ma$$
$$\boxed{T_1 = (\mu_k \cos\theta + \sin\theta)\, mg + ma}$$

From FBD II

$$\sum F_y = T_2 - Mg = -Ma$$
$$\boxed{T_2 = M(g-a)}$$

$$T_1 = T_2$$

$$\therefore\ (0.47 + 0.34)\ 17.6 + 1.8\,a = 19.6 - 2a$$
$$a = 1.4\ m/s^2$$