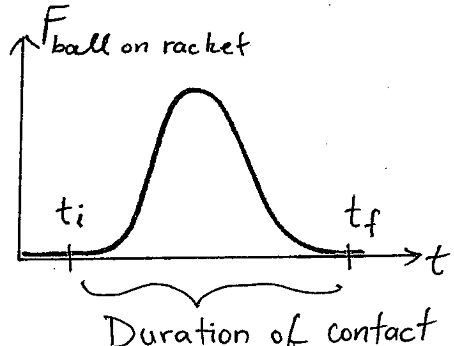
CHAPTER 9

Conservation Laws: A quantity that stays same throughout an interaction is said to be conserved. E.g., conservation of mass : $M_f = M_i$.

* Conservation Laws let us circumvent some complicated part of a problem and reduce the problem to "initial condition" vs. "final condition".

9.1 Momentum and Impulse

Consider a tennis ball colliding with a racket. To describe the motion, we need to analyze forces the ball and the racket exert each other. → Complicated, time-dependent forces at play.



$$ma_x = m\frac{dv_x}{dt} = F_x(t)$$

$$m\,dv_x = F_x(t)\,dt$$

$$m\int_{v_i}^{v_f} dv_x = \int_{t_i}^{t_f} F_x(t)\,dt$$

$$mv_f - mv_i = \int_{t_i}^{t_f} F_x(t)\,dt$$

Define Momentum → $\boxed{\vec{p} = m\vec{v}}$; $(p_x, p_y) = (mv_x, mv_y)$

$$p_{fx} - p_{ix} = \boxed{\Delta p_x = \int_{t_i}^{t_f} F_x(t)\,dt}$$

* Newton's 2nd Law was originally → $\boxed{\frac{d\vec{p}}{dt} = \frac{d(m\vec{v})}{dt} = \vec{F}(t)}$

* Unit of p is kgm/s

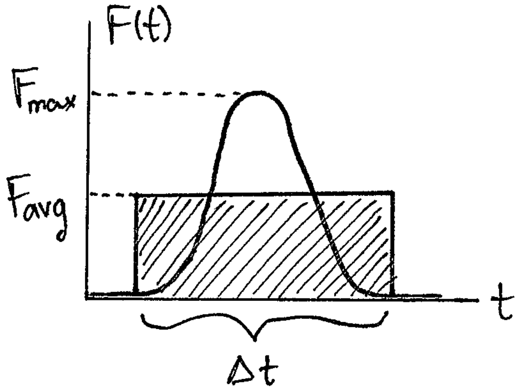
<u>For constant mass</u> : $m\frac{d\vec{v}}{dt} = \vec{F}(t)$

$\vec{F} = \dfrac{d\vec{p}}{dt}$ → Force is rate of change of momentum.

Define Impulse : $\boxed{J_x = \int_{t_i}^{t_f} F_x(t)\,dt}$ Area under $F_x(t)$ curve b/w t_i and t_f.

(Units : $Ns = \frac{kg\,m}{s}$)

We can also define impulse in terms of average force exerted in time interval $t_f - t_i = \Delta t$.



$$\boxed{J_x = F_{avg}\,\Delta t}$$

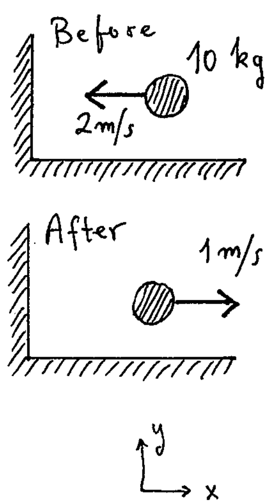
With the definitions of momentum & impulse we arrive at :

Impulse - Momentum Theorem :

$$\boxed{\Delta p_x = J_x}$$

An <u>impulse</u> delivered to a particle changes its <u>momentum</u>.

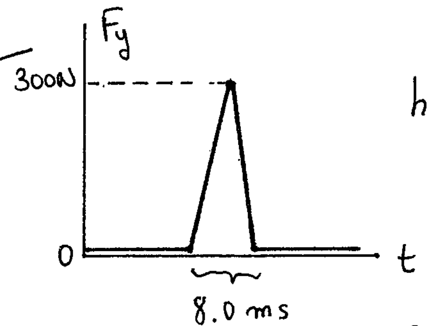
<u>Example</u>



What is the change in momentum?

$$\begin{aligned}
\Delta p_x &= p_{xf} - p_{xi} \\
&= m v_{xf} - m v_{xi} \\
&= 10\,kg\,(+1\ m/s) - 10\,kg\,(-2\ m/s) \\
&= 10\ \frac{kg\,m}{s} + 20\ \frac{kg\,m}{s} \\
&= +30\ kg\,m/s
\end{aligned}$$

PROB



A 100 g rubber ball is dropped from a height of 2.00 m onto a hard floor. How high does the ball bounce?

Ball's velocity immediately before collision:

$$v_{1y}^2 = v_{0y}^2 + 2a\Delta y$$

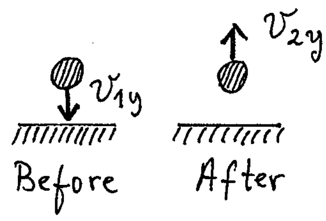
$$v_{1y}^2 = 2(-9.80\ \tfrac{m}{s^2})(-2.00\,m - 0.00\,m)$$

$$\boxed{v_{1y} = -6.26\ m/s}$$

From Impulse-Momentum Th.

$$mv_f - mv_i = J_x$$

$$\boxed{mv_{2y} - mv_{1y} = J_x}$$



J_x is Area under F_y curve;

$$J_x = \frac{300\,N\,(8.00\times10^{-3}\,s)}{2}$$

$$\boxed{J_x = 1.20\ Ns}$$

$$v_{2y} = v_{1y} + \frac{J_x}{m} = 5.74\ \frac{kgm}{s}$$

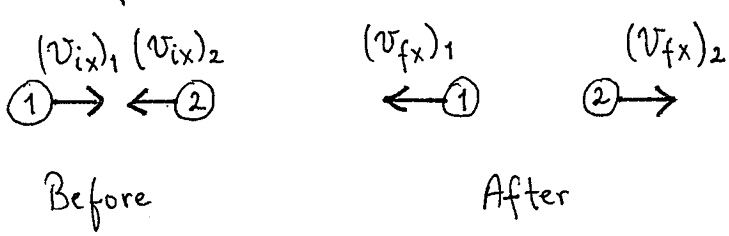
From Kinematics $\rightarrow$ $v_{3y}^2 = \boxed{0 = v_{2y}^2 - 2g\,\Delta y}$

$$\Delta y = 1.68\ m$$

9.3 Conservation of Momentum

* The impulse-momentum theorem is another way of looking at Newton's 2nd Law.

* Now incorporate Newton's 3rd Law



$$\frac{d(p_x)_1}{dt} = (F_x)_{2\text{ on }1}$$

$$\frac{d(p_x)_2}{dt} = (F_x)_{1\text{ on }2} = -(F_x)_{2\text{ on }1}$$

$$\frac{d(p_x)_1}{dt} = -\frac{d(p_x)_2}{dt}$$

$$\frac{d(p_x)_1}{dt} + \frac{d(p_x)_2}{dt} = 0$$

$$\frac{d}{dt}\left[(p_x)_1 + (p_x)_2\right] = 0$$

Therefore $\boxed{(p_x)_1 + (p_x)_2 = \text{constant}}$

$$(p_{fx})_1 + (p_{fx})_2 = (p_{ix})_1 + (p_{ix})_2$$

A more general approach for a system involving many interacting objects/particles:

Consider a system consisting of N particles in which particles interact via action/reation pairs $\rightarrow$ $\vec{F}_{j\text{ on }k} = -\vec{F}_{k\text{ on }j}$

The <u>total</u> momentum: $\vec{P} = \vec{p}_1 + \vec{p}_2 + \cdots + \vec{p}_N = \sum\limits_{k=1}^{N} \vec{p}_k$

Newton's 2nd Law: $\boxed{\dfrac{d\vec{P}}{dt} = \sum\limits_{k=1}^{N} \dfrac{d\vec{p}_k}{dt} = \sum\limits_{k=1}^{N} \vec{F}_k}$

Newton's 3rd Law: $\boxed{\vec{F}_k = \sum\limits_{j \neq k}^{N} \vec{F}_{j\text{ on }k} + \vec{F}_{\text{external on }k}}$

Since $\vec{F}_{j\text{ on }k}$ and $\vec{F}_{k\text{ on }j}$ are action/reaction pairs:

$\vec{F}_{j\text{ on }k} = -\vec{F}_{k\text{ on }j}$ $\rightarrow$ $\boxed{\vec{F}_{j\text{ on }k} + \vec{F}_{k\text{ on }j} = 0}$

Therefore $\boxed{\sum_k \sum_{j \neq k} \vec{F}_{j\,on\,k} = 0}$ Newton's 3rd Law

This results in

$$\boxed{\frac{d\vec{P}}{dt} = \sum_k \vec{F}_{ext\,on\,k} = \vec{F}_{net}}$$

This is same as Newton's 2nd law written for the entire system as a whole. The rate of change of the total momentum of a system is equal to the net force applied to the system.

* For an isolated system, $\vec{F}_{net} = \vec{0}$,

$$\frac{d\vec{P}}{dt} = 0$$

Law of Conservation of Momentum: The total momentum $\vec{P}$ of an isolated system does not change. $\vec{P}_f = \vec{P}_i$

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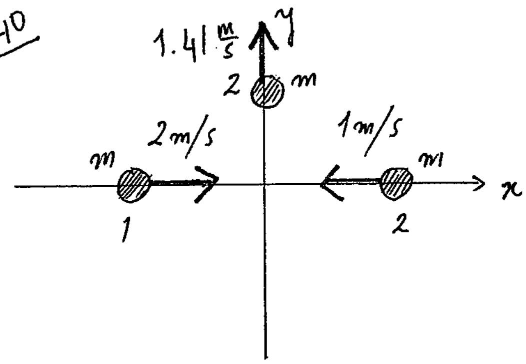


What is the velocity of the first ball after collision?

$$\begin{cases} P_{ix} = P_{fx} \\ P_{iy} = P_{fy} \end{cases}$$ Total momentum of the system is conserved.

$P_{ix} = P_{fx}$

$$m\,(2\ m/s) + m\,(-1\ m/s) = m\,(v_{fx})_1 + m\,(0\ m/s)$$

$$(v_{fx}) = 1\ m/s$$

$P_{iy} = P_{fy}$

$$m\,(0\ m/s) + m\,(0\ m/s) = m\,(v_{fy})_1 + m\,(1.41\ m/s)$$

$$(v_{fy})_1 = -1.41\ m/s$$

The speed of the ball #1 after collision is

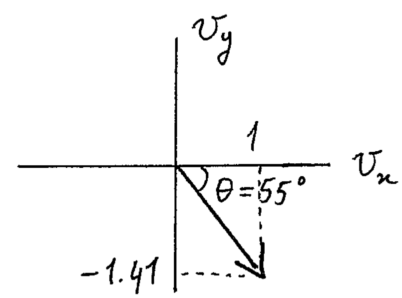
$$(v_f)_1 = \sqrt{(v_{fx})_1^2 + (v_{fy})_1^2}$$

$$= \sqrt{(1\ m/s)^2 + (-1.41\ m/s)^2}$$

$$= 1.7\ m/s$$

The direction of its motion is :

$$\theta = atan\left(\frac{(v_{fy})_1}{(v_{fx})_1}\right) = -55°$$

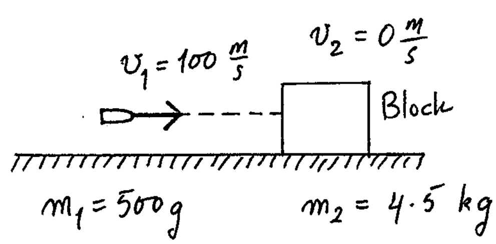


9.4 Inelastic Collisions

A collision in which the <u>two objects stick together and move with a common final velocity</u> is called a perfectly inelastic collision.

In an <u>elastic collision</u>, the two objects <u>bounce apart</u>. A full analysis requires conservation of energy. (Ch 10)

<u>PROB</u>



A bullet is shot at a wooden block as seen in the figure. It gets stuck inside the block and they move together afterwards. How much distance does the block & bullet traverse in 4 s?

$$P_{ix} = P_{fx}$$

$$m_1 v_1 + m_2 v_2 = (m_1 + m_2)\, v_3$$

$$50\ \frac{kg\,m}{s} + 0\ \frac{kg\,m}{s} = (5\ kg)\, v_3 \rightarrow \quad v_3 = 10\ m/s$$

The rest of the problem is Kinematics:

$$\Delta x = v_3 \Delta t + \frac{1}{2} a_x (\Delta t)^2 \quad \text{but } a_x = 0.$$

$$\Delta x = v_3 \Delta t$$

$$\Delta x = 10 \tfrac{m}{s} \cdot 4 \, s = 40 \, m$$

9.5 Explosions

In an explosion, the particles of the system move apart from each other after a brief + intense interaction. The explosive forces are internal forces and if the system is isolated, total momentum is conserved.

PROB 20 A 50 kg archer, standing on frictionless ice, shoots a 100 g arrow at a speed of 100 m/s. What is the recoil speed of the archer?

$$P_{ix} = P_{fx}$$

$$(m_{Arch} + m_{Arrow}) v_0 = m_{Arch} v_{Arch} + m_{Arrow} v_{Arrow}$$

$$50.1 \, kg \cdot 0 \tfrac{m}{s} = 50 \, kg \, v_{Arch} + 0.1 \, kg \cdot 100 \, m/s$$

$$0 = 50 \, kg \, v_{Arch} + 10 \, kg \, m/s$$

$$v_{Arch} = -\frac{1}{5} \, m/s = -0.2 \, m/s$$

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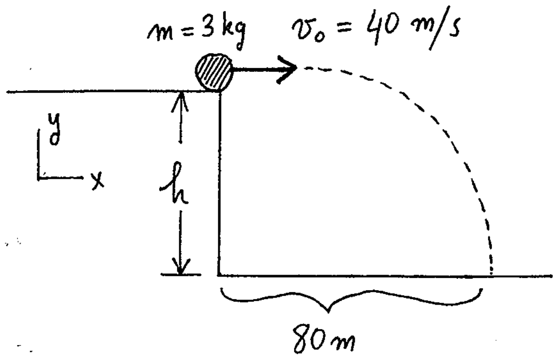


What is the impulse received by the object during the time it is in flight?

Time it takes for it to hit the ground Δt is,

$$\Delta x = v_0 \, \Delta t$$
$$80\ m = 40\ m/s \ \Delta t$$
$$\Delta t = 2\ s$$

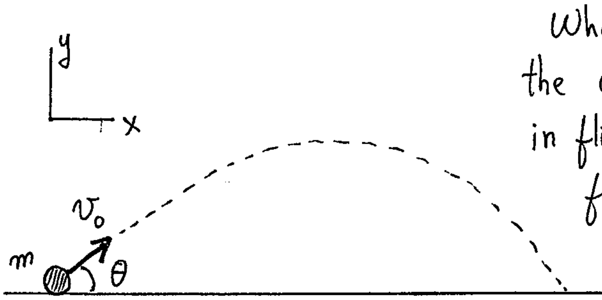
The vertical velocity v_{1y} acquired during Δt (flight) is

$$v_{1y} = v_{0y} - g\,\Delta t$$
$$v_{1y} = -9.80 \tfrac{m}{s^2} \cdot 2s = -19.6\ m/s$$

The impulse received is the change in momentum, $\Delta p_y = J_y$

$$\Delta p_y = m v_{1y} - m v_{0y} \quad , \quad \Delta p_x = 0 \cdot (a_x = 0) \rightarrow J_x = 0\ N \cdot s$$
$$\Delta p_y = 3\ kg\,(-19.6\ m/s) - 3\ kg \cdot 0\ m/s$$
$$\Delta p_y = 58.8\ kg\,m/s$$
$$J_y = 58.8\ kg\,m/s$$

PROB



What is the impulse received by the object during the time it is in flight? Deduce time of flight from Impulse-Momentum Th.

$$J_x = \Delta p_x = 0$$
$$J_y = \Delta p_y = m v_{yf} - m v_{yi}$$
$$\Delta p_y = m(-v_0 \sin\theta) - m(+v_0 \sin\theta)$$
$$\Delta p_y = -2 m v_0 \sin\theta$$

Also impulse is : $J_y = F_{net,y}\,\Delta t = -mg\,\Delta t$

From impulse-momentum theorem :

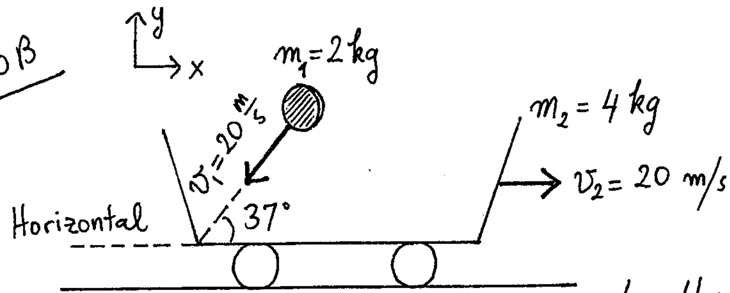
$$J_y = \Delta p_y$$

$$-mg\,\Delta t = -2\,m\,v_0 \sin\theta$$

$$\boxed{\Delta t = \frac{2 v_0 \sin\theta}{g}}$$

same as what we had from kinematics.

PROB



A 2kg object is thrown into a cart w/ 20 $\frac{m}{s}$ speed as seen in the figure. If they stick together after the collison, what is the speed of the cart after collision?

* Conservation of the x-component of the total momentum:

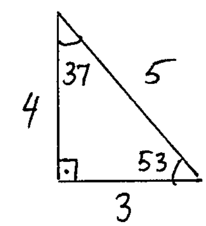
$$p_{ix} = p_{fx}$$

$$-m_1 v_1 \cos 37° + m_2 v_2 = (m_1 + m_2)\, v_{fx}$$

$$-2.0\,kg\ 20\,\frac{m}{s}\ 0.8 + 4.0\,kg\ 20\,\frac{m}{s} = 6\,kg\ v_{fx}$$

$$-32\,\frac{kg\,m}{s} + 80\,\frac{kg\,m}{s} = 6\,kg\ v_{fx}$$

$$v_{fx} = 8\ m/s$$



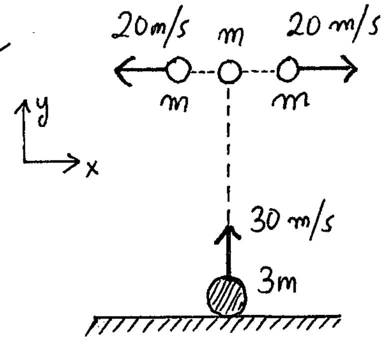
$$\cos 37° = \frac{4}{5} = 0$$

* We'll assume that the cart has shock absorbers.

* The y component of the total momentum is <u>not conserved</u>.

$$p_{iy} \neq p_{fy}$$

PROB



An object of mass 3m is shot straight up with a speed of 30 m/s. After 2s it was shot, it explodes and separates into 3 pieces with equal masses as seen in the figure. What is the speed and direction of the third fragment?

First we need the speed of the mass 3m <u>immediately before fragmentation</u>

$$V_{fy} = V_{iy} - g\,\Delta t$$

$$V_{fy} = 30\ m/s - 9.80\,\frac{m}{s^2} \cdot 2s \quad = 10.4\ m/s$$

* p_x is conserved by the two fragments w/ 20 m/s speed.

$$p_{xi} = p_{xf}$$
$$0 = m(-20\ m/s) + m(20\ m/s)$$
$$0 = 0$$

* Conservation for y-direction:

$$p_{yi} = p_{yf}$$
$$3\cancel{m}\, v_{fy} = \cancel{m}\, v_3$$
$$3\,(10.4\ \tfrac{m}{s}) = v_3$$
$$v_3 = 11.2\ m/s \text{ in the } +y \text{ direction.}$$