CHAPTER 11

* Questions we left unanswered in Ch. 10

    1) How many kinds of energy are there?
    2) Under what conditions is E conserved?
    3) How does a system lose or gain E?

* $\boxed{E_{mech} = K + U}$ → K and U can be changed into one another.

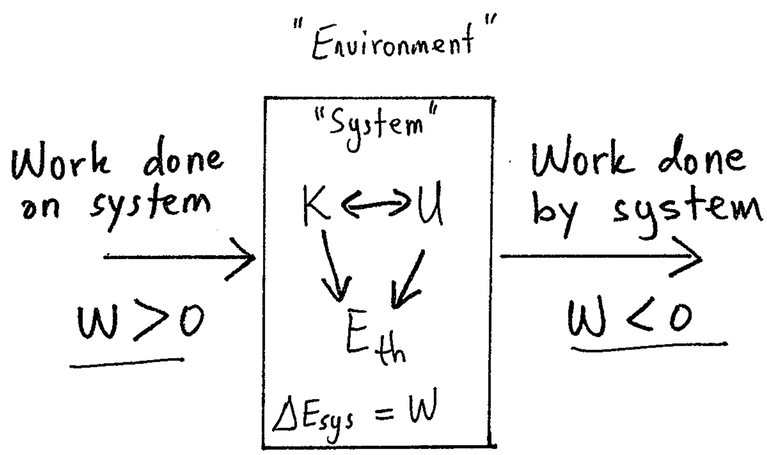
* Total energy of the moving atoms and stretched bonds inside an object is called object's thermal energy, E_{th}.

$$\boxed{E_{sys} = E_{mech} + E_{th} = K + U + E_{th}}$$ Total Energy of system

* K and U can also be changed into E_{th}.

* We'll be concerned by energy transfer due to pulls and pushes.

<u>Definition:</u> Mechanical transfer of energy to or from the system is called <u>work</u>, W.



$$E_{sys} = K + U + E_{th}$$

$$\boxed{\Delta E_{sys} = \Delta K + \Delta U + \Delta E_{th} = W}$$

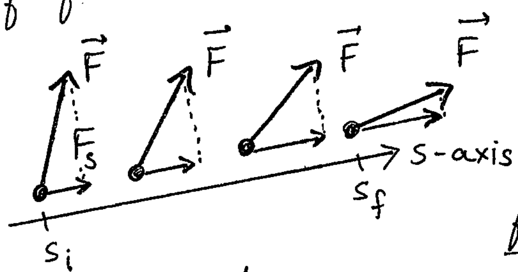
Two distinct cases

1) Energy can be transfered b/w system and environment
$$\boxed{\Delta E_{sys} = W}$$

2) Energy can be transfered b/w K, U and E_{th} within the system : $\boxed{\Delta E_{sys} = 0.}$

11.2 Work and Kinetic Energy

* The word "work" refers to something very specific in physics. Refrain from colloquial usage.
* Work is energy transfered to or from a system by application of force.



Consider a particle moving along s-axis alone.

Only F_s component of the applied force can change its speed.

$$F_s = \frac{dp_s}{dt} = m\frac{dv_s}{dt}$$

$$F_s = m\frac{dv_s}{ds}\underbrace{\frac{ds}{dt}}_{v_s} = m v_s \frac{dv_s}{ds}$$

$$\int_{s_i}^{s_f} F_s\, ds = m\int_{v_{si}}^{v_{sf}} v_s\, dv_s$$

$$\underbrace{\frac{1}{2}m v_{sf}^2 - \frac{1}{2}m v_{si}^2}_{=\Delta K} = \underbrace{\int_{s_i}^{s_f} F_s\, ds}_{\equiv W \;\rightarrow\; \text{Work}}$$

$$\boxed{\Delta K = W} \rightarrow \text{Work-Energy Theorem}$$

* Unit of work : $N.m = \frac{kg\,m}{s^2}m = \frac{kg\,m^2}{s^2} = \text{Joule} \equiv J$

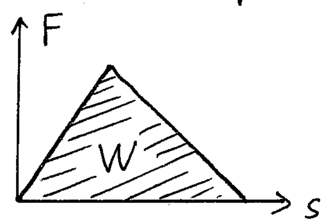
$$\boxed{W = \int_{s_i}^{s_f} F_s \, ds} \rightarrow \text{No displacement means no work!}$$

* For a system acted upon by many forces $\vec{F}_1, \vec{F}_2, \ldots$

$\vec{F}_{net} = \sum_i \vec{F}_i$ therefore

$$\boxed{W_{net} = \sum_i W_i} \quad \text{because} \quad W_{net} = \int_{s_i}^{s_f} F_{net,s} \, ds$$

* Work is also interpreted as the area under F-s curve.

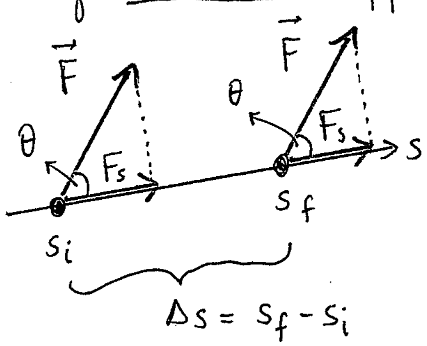


* Kinetic Energy can also be written in terms of momentum:

$$\boxed{K = \frac{1}{2} m v^2 = \frac{p^2}{2m}}$$

$$\begin{cases} \Delta K = W = \int_{s_i}^{s_f} F_s \, ds \\ \Delta p_s = J_s = \int_{t_i}^{t_f} F_s \, dt \end{cases}$$

Case of <u>constant</u> Applied Force



$$W = \int_{s_i}^{s_f} F_s \, ds = \int_{s_i}^{s_f} F \cos\theta \, ds$$

$$W = F \cos\theta \underbrace{\int_{s_i}^{s_f} ds}_{= \Delta s} = \boxed{F \cos\theta \, \Delta s}$$

$$\boxed{W = F\cos\theta\,\Delta s}$$

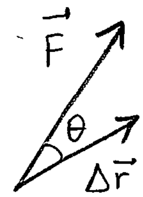
→ If the force is perpendicular to the direction of motion (direction of $\vec{v}$), $W = 0$.

→ If $\Delta s = 0$ then $W = 0$.

* We can express W in terms of the dot product of vectors $\vec{F}$ and $\Delta\vec{r}$.

$$\boxed{W = \vec{F}\cdot\Delta\vec{r} = F\,\Delta r\,\cos\theta}$$

Projection of $\vec{F}$ onto $\Delta\vec{r}$.



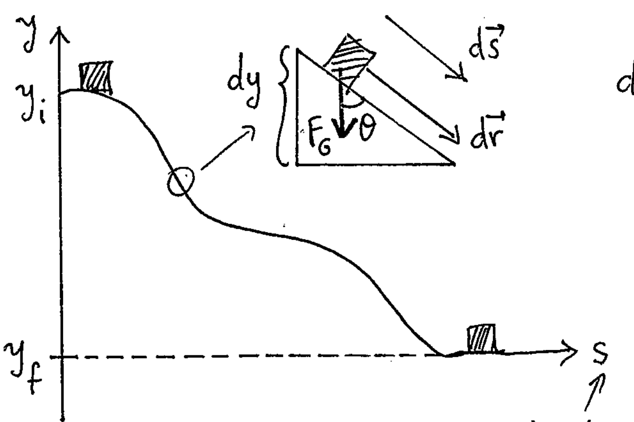
* Dot product of two vectors:

$$\left.\begin{aligned}\vec{A} &= A_x\,\hat{\imath} + A_y\,\hat{\jmath}\\ \vec{B} &= B_x\,\hat{\imath} + B_y\,\hat{\jmath}\end{aligned}\right\}\quad \begin{aligned}\vec{A}\cdot\vec{B} &= A_x B_x + A_y B_y\\ \vec{A}\cdot\vec{B} &= \vec{B}\cdot\vec{A}\\ \vec{A}\cdot\vec{A} &= |\vec{A}|^2 = A_x^2 + A_y^2\end{aligned}$$

* When the force $\vec{F}$ is not constant:

$$\boxed{W = \int_{\vec{r}_i}^{\vec{r}_f} \vec{F}\cdot d\vec{r}}$$

11.5 Force, Work and Potential Energy



We chose s-axis in the direction of motion.

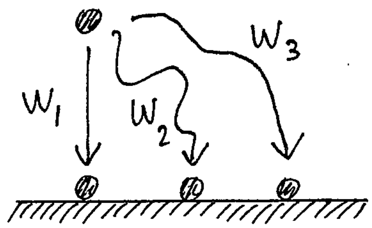
$$dy = -\cos\theta\,ds$$

$$dW_{grav} = \vec{F}_G\cdot d\vec{r} = mg\underbrace{\cos\theta\,ds}_{-dy}$$

$$dW_{grav} = -mg\,dy$$

$$\boxed{W_{grav} = -mg\,\Delta y}$$

* The work done by gravity is independent of the path the object takes to get from s_i to s_f.

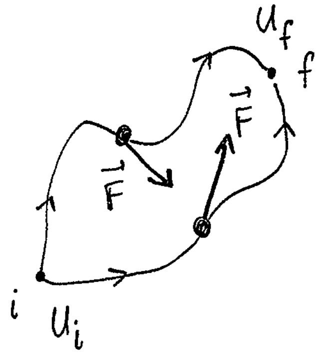


$$W_1 = W_2 = W_3$$

Conservative and Nonconservative Forces

* A force for which work done on a system is independent of the path taken is called a conservative fore. E.g., gravity.
* A potential energy can be associated with any conservative force!

Work-energy th.: $\left.\begin{array}{l}\Delta K = W \\ \Delta K = -\Delta U\end{array}\right\}$ $\begin{array}{l}\Delta U = -W \\ U_f - U_i = -W(i \to f)\end{array}$



* F_G and F_{spring} are conservative forces.

$$U_f - U_i = -W(i \to f) = -(mgy_i - mgy_f)$$

$$U_f - U_i = mgy_f - mgy_i$$

* Not all forces are conservative. E.g. friction.

$$dW_{fric} = \vec{f}_k \cdot d\vec{s} = -\mu_k mg\, ds$$

$$W_{fric} = -\mu_k mg \underbrace{\Delta s}_{\text{the total distance}}.$$

* There is no potential of friction!