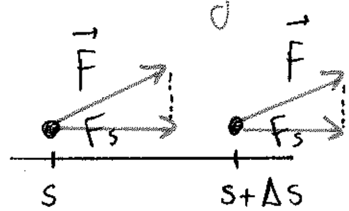
11.6 Finding Force From Potential Energy



The work done on the object is

$$W(s \rightarrow s+\Delta s) = F_s \Delta s$$

If $\vec{F}$ is a conservative force,

$$\Delta U = -W(s \rightarrow s+\Delta s) = -F_s \Delta s$$

$$F_s = -\frac{\Delta U}{\Delta s}$$

For a path of arbitrary shape,

$$\boxed{F_s = -\lim_{\Delta s \to 0} \frac{\Delta U}{\Delta s} = -\frac{dU}{ds}}$$

* E.g., Gravitational Force $F_G = -\frac{dU_g}{dy} = -\frac{d}{dy}(mgy)$

$$F_G = -mg$$

* F_s is the negative of the slope of the U vs. s graph.

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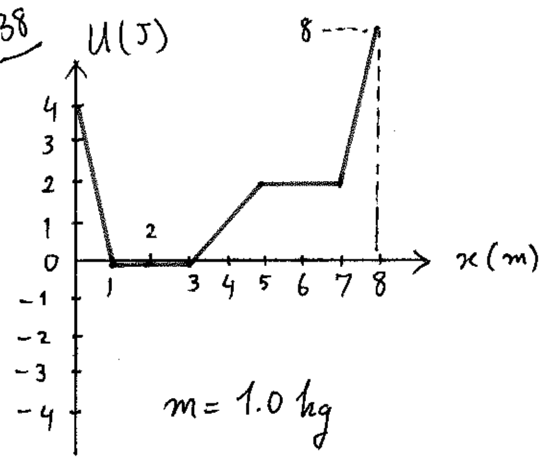
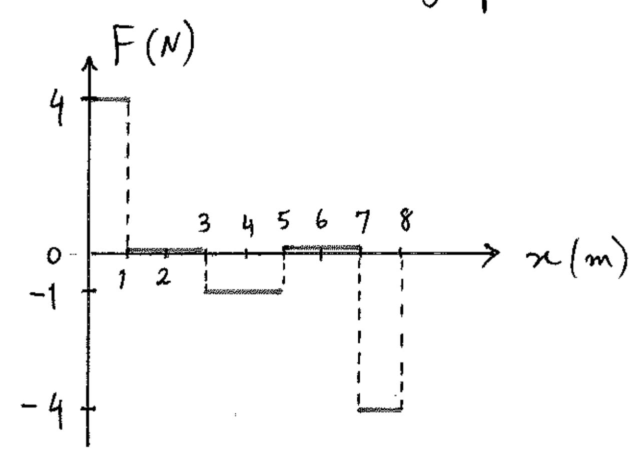


a) Draw F vs. x graph.



b) How much work does the Force do as the particle moves from $x = 2\ m$ to $x = 6\ m$?

$$W(x=2m \rightarrow x=6m) = \left(\begin{array}{c}\text{Area under } F-x \text{ curve between} \\ x=2m \text{ and } x=6m\end{array}\right)$$

$$W = (-1\ N)(2m)$$

$$W = -2\ J.$$

c) What speed the particle needs at $x=2m$ to arrive at $x = 6\ m$ with a speed of $10\ m/s$?

Work - Energy theorem: $\boxed{W = \Delta K}$

$$W(x=2m \rightarrow x=6m) = K(@x=6m) - K(@x=2m)$$

$$-2\ J = \frac{1}{2} m\,(10\ m/s)^2 - \frac{1}{2} m\, v_0^2$$

$$-2\ J = 50\ \frac{kg\, m^2}{s^2} - 0.5\ kg\ v_0^2$$

$$0.5\ kg\ v_0^2 = 52\ \frac{kg\, m^2}{s^2}$$

$$v_0^2 = 104\ \frac{m^2}{s^2}$$

$$v_0 = 10.2\ m/s$$

11.9 Power

Work is energy transferred between environment and system.
Power is how quickly that energy is transferred.

$$\boxed{P \equiv \frac{dE_{sys}}{dt}} \rightarrow \text{Rate of change of total Energy of the system.}$$

Unit of P in SI is $\boxed{\text{Watt} = W = J/s.}$

* Power as "rate of doing work"

$$P = \frac{dW}{dt} \quad \text{and} \quad dW = \vec{F} \cdot d\vec{r}$$

$$\frac{dW}{dt} = \vec{F} \cdot \underbrace{\frac{d\vec{r}}{dt}}_{=\vec{v}} \quad \text{(For constant force)}$$

$$P = \vec{F} \cdot \vec{v}$$

$$P = F v \cos\theta$$

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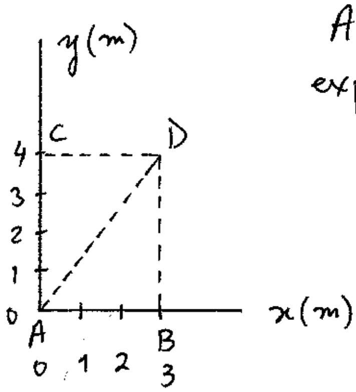


A particle moves from A to D. while experiencing force $\vec{F} = (6\hat{i} + 8\hat{j})$ N.

How much work is done on the particle if it follows

a) ABD?

b) ACD?

c) AD?

a) Path ABD $\rightarrow$ $\boxed{W_{ABD} = W_{AB} + W_{BD}}$

$$\boxed{= \vec{F} \cdot \Delta\vec{r}_{AB} + \vec{F} \cdot \Delta\vec{r}_{BD}}$$

$$= (6\hat{i} + 8\hat{j}) \cdot 3\hat{i} + (6\hat{i} + 8\hat{j}) \cdot 4\hat{j}$$

$$= 18 + 32 = 50 \text{ J}.$$

b) Path ACD $\rightarrow$ $\boxed{W_{ACD} = W_{AC} + W_{CD}}$

$$\boxed{= \vec{F} \cdot \Delta\vec{r}_{AC} + \vec{F} \cdot \Delta\vec{r}_{CD}}$$

$$= (6\hat{i} + 8\hat{j}) \cdot 4\hat{j} + (6\hat{i} + 8\hat{j}) \cdot 3\hat{i}$$

$$= 50 \text{ J}$$

c) Path AD $\rightarrow$ $\boxed{W_{AD} = \vec{F} \cdot \Delta\vec{r}_{AD}}$

$$= (6\hat{i} + 8\hat{j}) \cdot (3\hat{i} + 4\hat{j})$$

$$= 18 + 32 = 50 \text{ J}$$

The work $\vec{F}$ does <u>does not</u> depend on path.
Therefore $\vec{F}$ is a conservative force.

Curl of $\vec{F}$: $\vec{\nabla} \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 6 & 8 & 0 \end{vmatrix} = \hat{i} \cdot 0 + \hat{j} \cdot 0 + \hat{k} \cdot 0$

$= 0 \rightarrow \vec{F}$ is conservative

$\vec{F}$ is conservative if

1) $\vec{\nabla} \times \vec{F} = 0$

2) $W = \oint_c \vec{F} \cdot d\vec{r} = 0$

3) $\vec{F} = -\vec{\nabla} U$

PROB 59 $\quad F_x = -\gamma x^3$

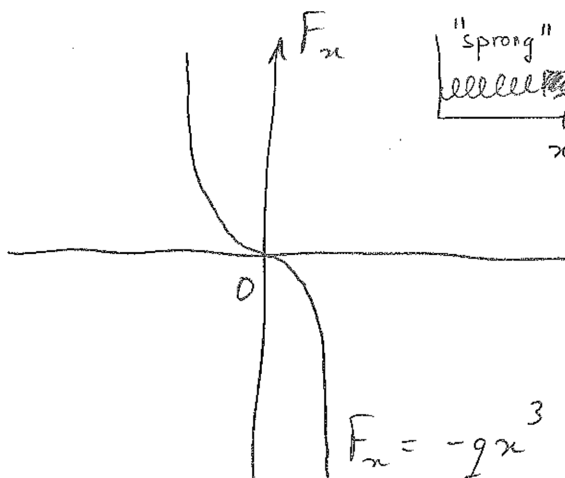
ⓐ Units of γ?
ⓑ Graph F_x
ⓒ What is U corresponding to F_x?
ⓓ $\gamma = 40{,}000 \frac{N}{m^3}$, 20 g marble shot w/ this device. What is v_f if $\Delta x = 10$ cm?

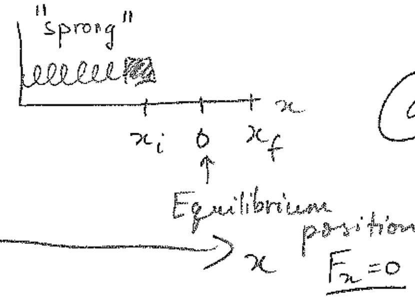
ⓒ $$\int_{x_i}^{x_f} -\gamma x^3\,dx = -\gamma\left(\frac{x^4}{4}+c\right)\Bigg|_{x_i}^{x_f} = -\gamma\left(\frac{x_f^4}{4} - \frac{x_i^4}{4}\right) = -\Delta U = -(U_f - U_i)$$

$$\Rightarrow U_f = \gamma\frac{x_f^4}{4}, \quad U_i = \gamma\frac{x_i^4}{4} \quad \Big\}\ U(x) = \gamma\frac{x^4}{4}$$

F_x is a conservative force $\leftarrow$

ⓑ





ⓐ $$[\gamma] = \frac{N}{m^3} = \frac{\frac{kg\,m}{s^2}}{m^3}$$

$$[\gamma] = \frac{kg}{m^2 s^2}$$

ⓓ Since F_x is conservative, I can choose my system to be composed of the marble and the sprong.

$$E_{sys} = K + U \quad \text{and} \quad \Delta E_{sys} = 0$$

$$K_i + U_i = K_f + U_f$$

$$\cancel{\tfrac{1}{2} m v_i^2}^{\,0} + \frac{1}{4}\gamma x_i^4 = \frac{1}{2} m v_f^2$$

$$v_f^2 = \frac{\gamma}{2m} x_i^4$$

$$v_f = \sqrt{\frac{\gamma}{2m}}\, x_i^2 = \sqrt{\frac{40{,}000 \frac{kg}{m^2 s^2}}{2\,(0.020\,kg)}}\,(0.1\,m)^2$$

$$v_f = 1000 \frac{1}{m \cdot s}\,(0.01\,m^2) = 10\ m/s$$

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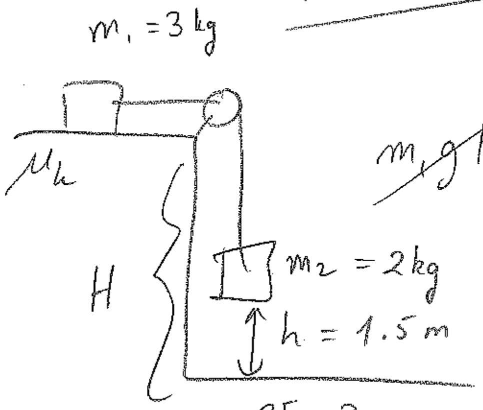
Release from Rest



a) No friction

$$\cancel{m_1 g H} + m_2 g h = \cancel{m_1 g H} + \frac{1}{2}(m_1+m_2)v_f^2$$

$$m_2 g h = \frac{1}{2}(m_1+m_2)v_f^2$$

$$v_f = \sqrt{\frac{2 m_2 g h}{m_1+m_2}} = \sqrt{\frac{2(2\,kg)(9.8\,\frac{m}{s^2})1.5\,m}{5\,kg}}$$

$$v_f = 3.4\ \frac{m}{s}$$

b) Friction $\mu_k = 0.15$

$$\cancel{m_1 g H} + m_2 g h = \cancel{m_1 g H} + \frac{1}{2}(m_1+m_2)v_f^2 + W$$

W = Work done by f_s

$$m_2 g h = \frac{1}{2}(m_1+m_2)v_f^2 + f_s \cdot h \quad , \quad f_s = m_1 g \mu_k = n\mu_k$$

FBD of m_1

$$\Sigma F_y = n - mg = 0$$
$$\boxed{n = mg}$$

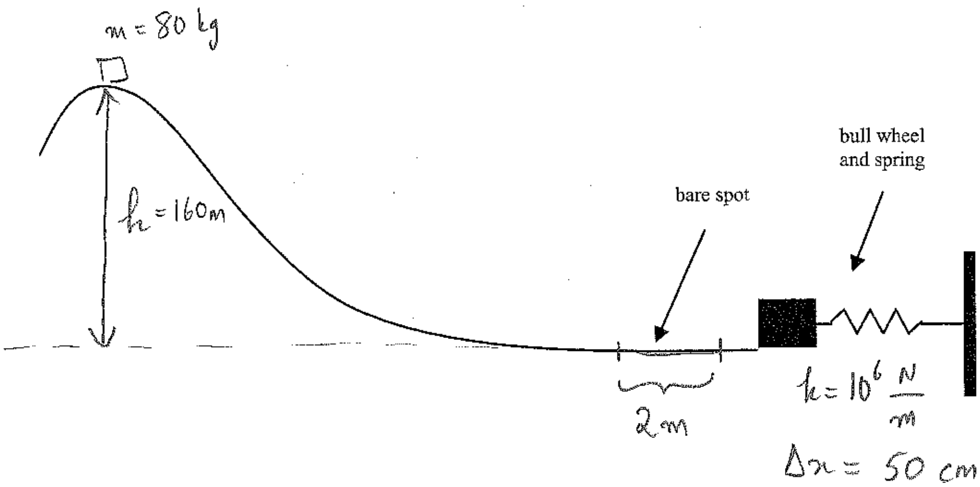
$$m_2 g h = \frac{1}{2}(m_1+m_2)v_f^2 + m_1 g \mu_k h$$

$$v_f = \sqrt{\frac{2}{m_1+m_2}(m_2 g h - m_1 g \mu_k h)}$$

$$v_f = \sqrt{\frac{2gh(m_2 - m_1\mu_k)}{m_1+m_2}} = \sqrt{\frac{2(9.8\,\frac{m}{s^2})1.5\,m(2\,kg - 3 \cdot 0.15\,kg)}{5\,kg}}$$

$$v_f = 3.0\ \frac{m}{s}$$

Example 11. This time our hapless 80 kg skier starts from rest at the top of a 160 meter tall hill. The downhill portion of the run is snow covered and essentially frictionless but a flat section at the base of the hill has a bare spot 2 meters in length. The skier skis straight down the hill, over the bare spot at the bottom, hits the counterweight on the bullwheel compressing the damping spring 50 centimeters in the process. If the spring constant is 1×10^6 N/m, what is the coefficient of kinetic friction between rental skis and bluegrass?



A non-conservative force, friction, is present here, $E_i \neq E_f$, and total mechanical energy is not conserved.

$E_i = mgh = (80\text{ kg})(9.8\text{ m/s}^2)(160\text{ m}) = 125440\text{ J}$
$E_f = \frac{1}{2}kx^2 = (0.5)(1 \times 10^6\text{ N/m})(0.5\text{ m}^2) = 125000\text{ J}$

$$W_f = \Delta E = -440J = \vec{f}_k \cdot \vec{s} = -\mu_k mgs$$

$$\frac{440J}{(2meters)(80kg)(9.8m \cdot s^{-2})} = \mu_k = 0.28$$

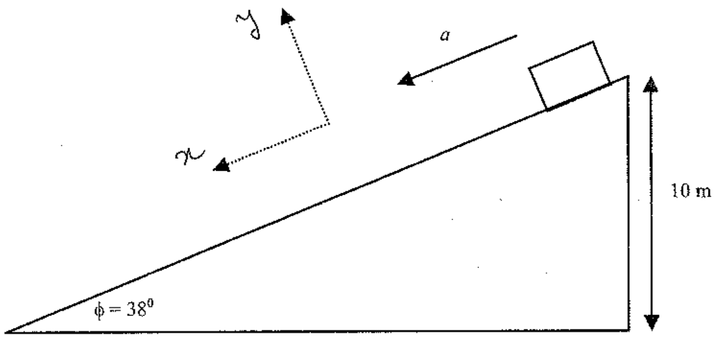
$$\cancel{K_i}^{0} + U_{g_i} = \cancel{K_f}^{0} + \cancel{U_{g_f}}^{0} + U_{s_f} + W$$

$$\underset{80\,kg}{m}\,\underset{160\,m}{g h} = \frac{1}{2}\underset{10^6\frac{N}{m}}{k}\,(\underbrace{\Delta x}_{50cm})^2 + \underset{?}{\mu_k}\,\underset{80\,kg}{m}g\,\underbrace{\Delta s}_{2.0m}\,, \quad W_{f_s} = \mu_k mg\,\Delta s$$

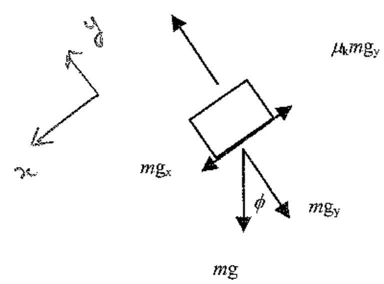
$$\mu_k = 0.28$$

Example 5: A 10 kg block slides down a rough surface (μ_k=0.2). It is released from rest at the top of an incline (38^0 to the horizontal) at a height 10 meters. What is its speed at the bottom of the incline?

We've seen this problem before as well.



Note: $\sin 38^0 = \frac{10m}{hyp} \Rightarrow$ the length of the incline is 16.2 meters. In the coordinate system we've chosen, the FBD is:



$\sum F_x = mg\sin\phi - f_s = ma$

$\sum F_y = -mg\cos\phi + n = 0$

$n = mg\cos\phi$

$f_s = \mu_k n = \mu_k mg\cos\phi$

Based on this FBD: $\sum F_x = mg\sin\phi - \mu_k mg\cos\phi = ma \therefore a = g\sin\phi - \mu_k g\cos\phi$ Acceleration of the block

The first term is the same expression as we got when working this example without accounting for the presence of friction. The second term is a component that reduces the overall acceleration due to the presence of kinetic friction.

In this case:

$$a = (9.8m\cdot s^{-2})(0.616) - (0.2)(9.8m\cdot s^{-2})(0.788) = (6.03m\cdot s^{-2}) - (1.54m\cdot s^{-2}) = 4.5m\cdot s^{-2}$$

By determining the acceleration we've solved the *dynamics* part of the problem and with this information we can proceed to solve the *kinematics* part of the problem.

$$v^2 = v_0^{\,2} + 2a(x - x_0) \rightarrow v = \sqrt{2a(x-x_0)} = \sqrt{(2)(4.5m\cdot s^{-2})(16.2m)} = 12.1m\cdot s^{-1}$$

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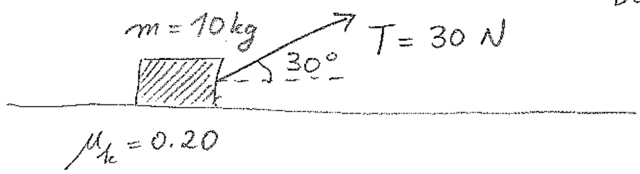
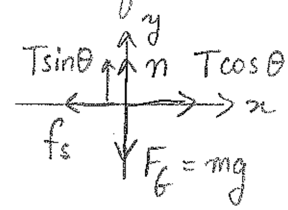
What is block's speed after being pulled 3.0 m?



FBD of block



(a)

$$\sum F_x = T\cos\theta - f_s = ma$$

$$\sum F_y = n + T\sin\theta - mg = 0$$

$$n = mg - T\sin\theta$$

$$f_s = \mu_k n = \mu_k (mg - T\sin\theta)$$

From $\sum F_x \rightarrow$ $a = \frac{1}{m}\left(T\cos\theta - f_s\right)$

$$a = \frac{1}{m}\left(T\cos\theta - \mu_k mg + \mu_k T\sin\theta\right)$$

$$a = \frac{1}{10\,kg}\left(30\,N * 0.87 - 0.20 * 10 * 9.8\,\frac{kg\,m}{s^2} + 0.20 * 30\,N * 0.5\right)$$

$$a = \frac{1}{10\,kg}\left(26\,N - 20\,N + 3\,N\right)$$

$$a = \frac{9\,N}{10\,kg} = 0.9\,\frac{m}{s^2}$$

From Kinematics $\longrightarrow$ $v^2 = 2a\Delta x$

$$v^2 = 2\left(0.9\,\frac{m}{s^2}\right) 3.0\,m$$

$$v^2 = 5.4$$

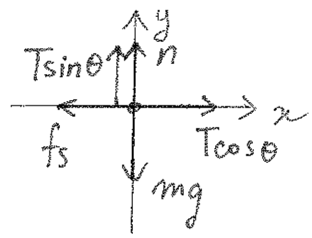
$$v = 2.3\,m/s$$

(b) Using the Work-Energy theorem,

$$\Delta K = W$$

$$\frac{1}{2} m v^2 - 0 = F \Delta x \qquad \text{where } F = (F_{net})_x$$

Draw FBD for block →



Not to apply Newton's Laws but to get $(F_{net})_x$

$$\sum F_x = T\cos\theta - f_s$$

$$\sum F_y = n + T\sin\theta - mg = 0$$

$$f_s = \mu_k n = \mu_k (mg - T\sin\theta)$$

$$(F_{net})_x = \sum F_x = T\cos\theta - \mu_k (mg - T\sin\theta)$$

Then $\Delta K = W$ becomes,

$$\frac{1}{2} m v^2 = \left[T\cos\theta - \mu_k (mg - T\sin\theta) \right] \Delta x$$

$$\frac{1}{2}\, 10\,\mathrm{kg}\, v^2 = \left[30\,\mathrm{N} * \cos 30^\circ - 0.20 \left(10\,\mathrm{kg} \cdot 9.8\,\frac{\mathrm{m}}{\mathrm{s}^2} - 30\,\mathrm{N} \sin 30^\circ \right) \right] 3.0\,\mathrm{m}$$

$$5\,\mathrm{kg}\, v^2 = (26\,\mathrm{N} - 17\,\mathrm{N}) \cdot 3.0\,\mathrm{m}$$

$$v^2 = 5.4$$

$$v = 2.3\,\frac{\mathrm{m}}{\mathrm{s}}$$