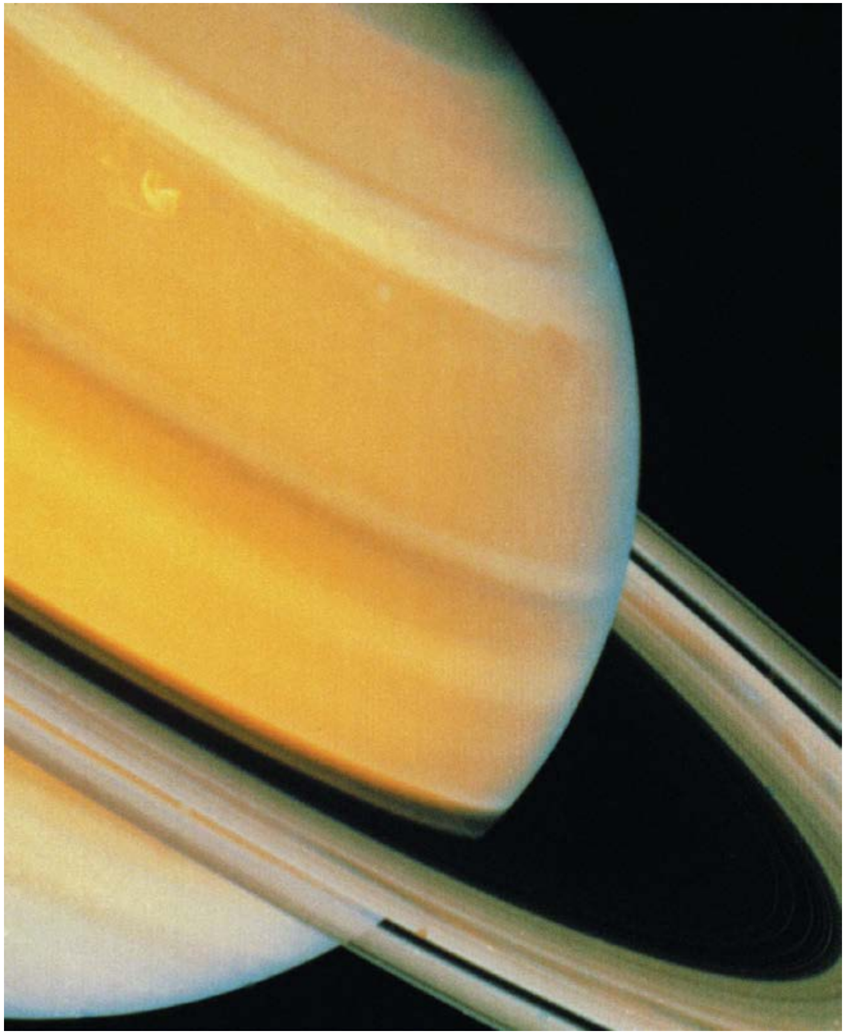
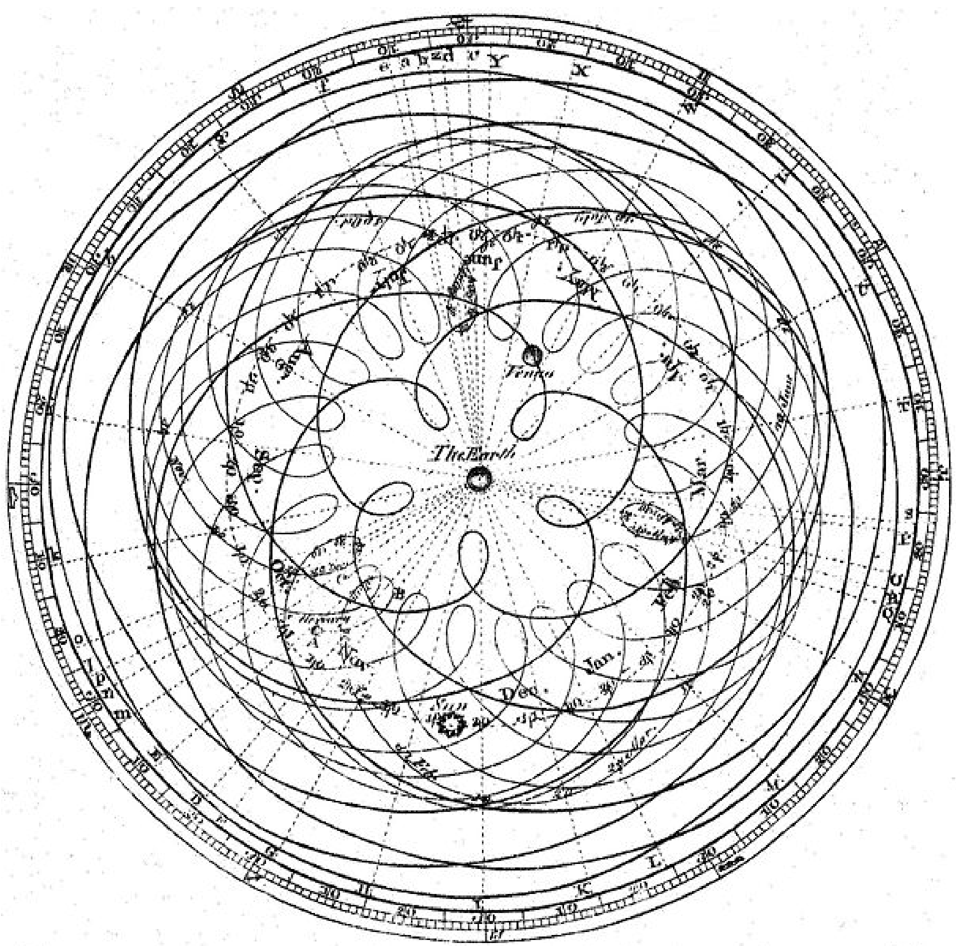
Chapter 13. Newton's Theory of Gravity

Chapter Goal: To use Newton's theory of gravity to understand the motion of satellites and planets.



Geocentric Model of Ptolemy

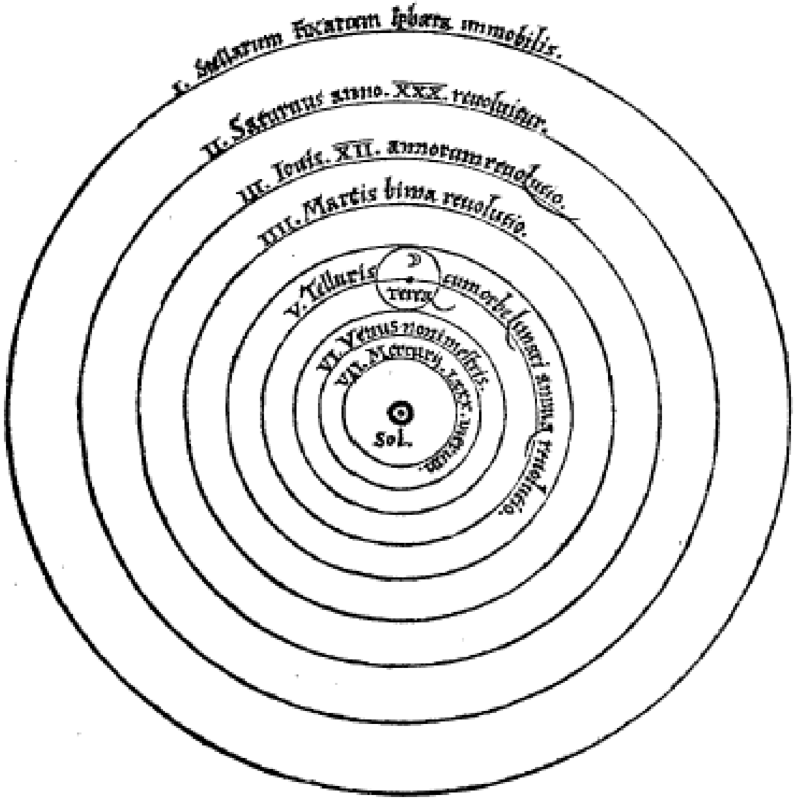


Earth at the center of the universe

From ancient Greeks to middle ages in Europe

Epicycles as orbits

Copernicus (circa 1543)



Copernicus adopted a model with sun at the center

Now the orbits were circular

Tycho and Kepler

Between 1570 – 1600, Tycho compiled most accurate astronomical observations known to that date

Tycho's young assistant Johannes Kepler analyzed the data for many years and made Three key observations.

Kepler’s Laws

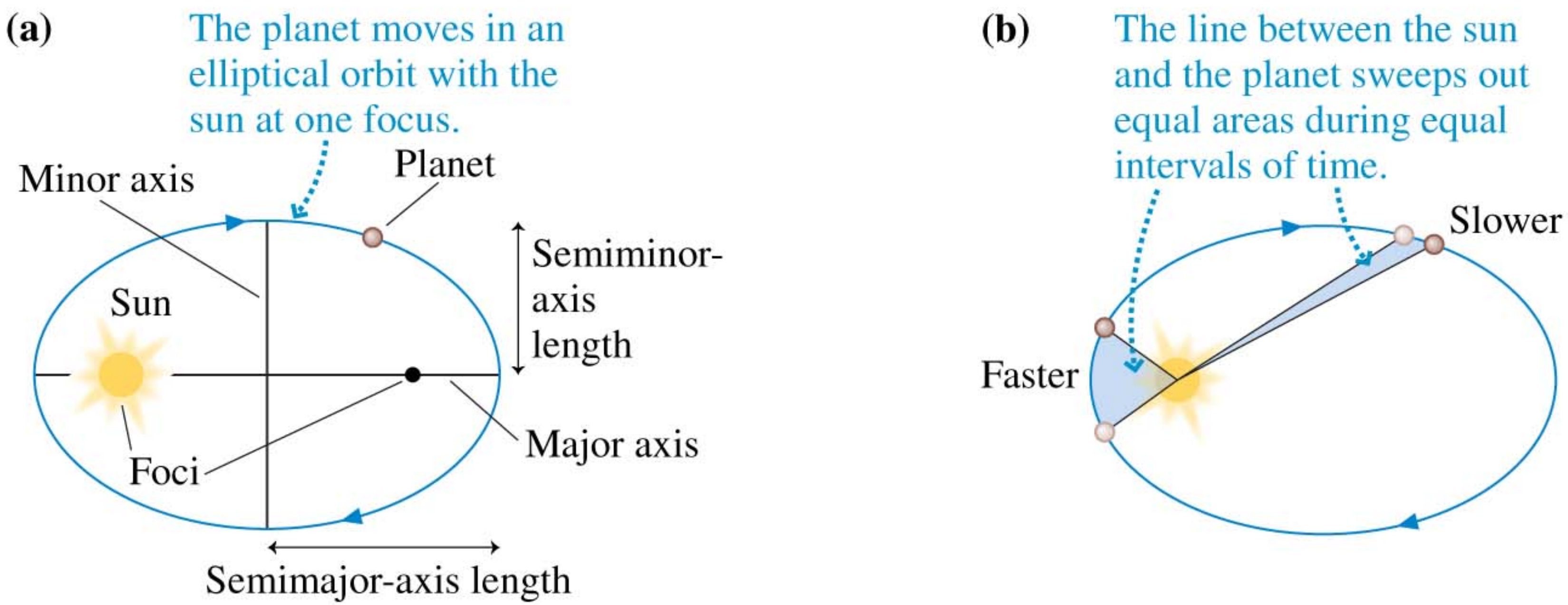
1. Planets move in elliptical orbits, with the sun at one focus of the ellipse.

2. A line drawn between the sun and a planet sweeps out equal areas during equal intervals of time.

3. The square of a planet’s orbital period is proportional to the cube of the semimajor-axis length.

Ellipses

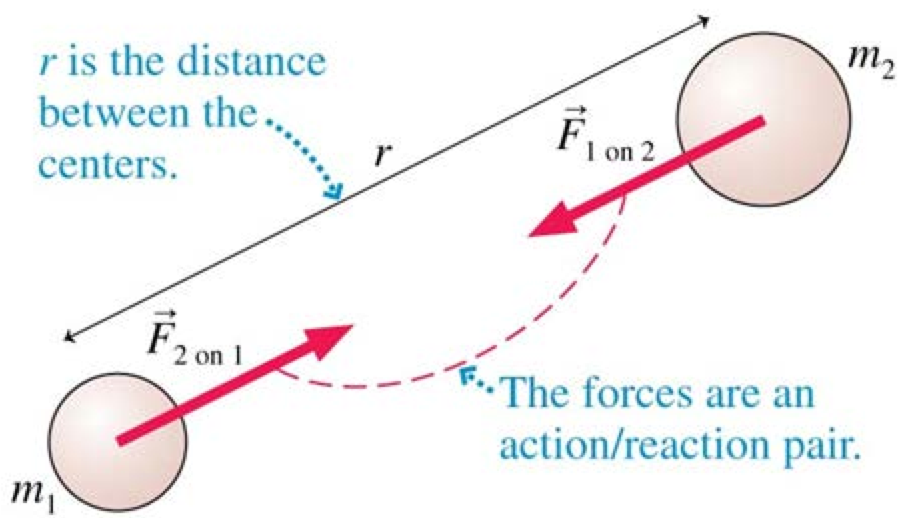
FIGURE 13.2 The elliptical orbit of a planet about the sun.



Newton’s Law of Gravity

Newton’s key contribution was the realization that the force of the sun on the planets was **identical** to the force of the earth on an apple.

Newton proposed that *every* object in the universe attracts *every other* object.



Newton's Law of Gravity

Newton's law of gravity If two objects with masses m_1 and m_2 are a distance r apart, the objects exert attractive forces on each other of magnitude

$$F_{1\text{ on }2} = F_{2\text{ on }1} = \frac{Gm_1m_2}{r^2} \qquad (13.2)$$

The forces are directed along the straight line joining the two objects.

The constant *G*, called the **gravitational constant**.

In the SI system of units, *G* has the value
$6.67 \quad 10^{-11}$ N m^2/kg^2.

Little g and Big G

Suppose an object of mass *m* is on the surface of a planet of mass *M* and radius *R*. The local gravitational force may be written as

$$F_G = mg_{\text{surface}}$$

where we have used a local constant acceleration:

$$g_{\text{surface}} = \frac{GM}{R^2}$$

On earth near sea level it can be shown that g_{surface} = 9.80 m/s^2.

Gravitational Potential Energy

When two isolated masses m_1 and m_2 interact over large distances, they have a gravitational potential energy of

$$U_g = -\frac{Gm_1m_2}{r}$$

where we have chosen the zero point of potential energy at $r = \infty$, where the masses will have no tendency, or potential, to move together.

Note that this equation gives the potential energy of masses m_1 and m_2 when their *centers* are separated by a distance r.

Orbital Energetics

We know that for a satellite in a circular orbit, its speed is related to the size of its orbit by $v^2 = GM/r$. The satellite's kinetic energy is thus

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$$

But $-GMm/r$ is the potential energy, U_g, so

$$K = -\frac{1}{2}U_g$$

If K and U do not have this relationship, then the trajectory will be elliptical rather than circular. So, the mechanical energy of a satellite in a circular orbit is always:

$$E_{mech} = K + U_g = \frac{1}{2}U_g$$