Generation of maximally entangled GHZ (Greenberger-Horne-Zeilinger) states of divalent atoms

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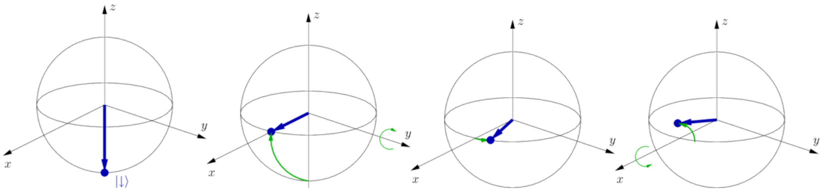
Oct 24, 2014

Table of Contents

Ramsey Spectroscopy (single atom)



1. Prepare atom in $|g\rangle$
2. A $\pi/2$-pulse rotates the state vector around y-axis
3. Free evolution (interrogation) time T. Accumulate phase $\delta\omega = (\omega - \omega_0)T$.
4. A second $\pi/2$-pulse rotates state vector around x-axis
   (it could be the same y-axis - it will still work the same way)
5. Measure whether the atom is in the $|g\rangle$ or $|e\rangle$ state. Depending on whether $\delta\omega$ was positive or negative, the probability find the atom in either the upper or the lower state is larger. Adjust the frequency to compensate.

Clock stability

- Frequency uncertainty from a single measurement of N 2-level atoms with collective spin vector $\mathbf{J} = \sum_n^N \mathbf{J}_n$:

$$\delta\phi = \frac{\Delta J_z(\phi)}{|\partial\langle\Delta J_z(\phi)\rangle/\partial\phi|}$$

- $\delta\phi$ can be minimized either by minimizing the projection noise (spin squeezing) OR maximizing the signal slope (GHZ states).
- For "**uncorrelated**" atoms: $\delta\phi \geq \frac{1}{\sqrt{N}}$
  (Called the Standard Quantum limit (SQL).)
- For "**correlated**" atoms: $\delta\phi \geq \frac{1}{N}$
  (Called the Heisenberg limit.)

See, for example, A. Andre and M. D. Lukin PRA **65**, 053819 (2002), A. Andre, "Nonclassical states of light and atomic ensembles: Generation and New Applications", PhD thesis, Harvard University, Cambridge, Massachusetts (2005), A. D. Ludlow *et al.*, arXiv:1407.3493 [physics.atom-ph]

Correlated versus *Uncorrelated*

- The motivation is to beat the Standard quantum limit (SQL) and get closer to the Heisenberg limit in a Ramsey spectroscopic scheme for improved atomic clock stability.

  1. With Ramsey spectroscopy using N **uncorrelated** atoms, the minimal attainable phase sensitivity is $1/\sqrt{N}$ (SQL).
  2. This limit can be beaten by introducing correlations between the atoms : **Assume, after the first $\pi/2$-pulse, we have the state (GHZ)**

$$|\psi_M\rangle = \frac{1}{\sqrt{2}}\left(|ggg\cdots g\rangle + |eee\cdots e\rangle\right)$$

  Note that this state cannot be prepared by the first $\pi/2$-pulse in the Ramsey sequence from $\sum_i |g\rangle_i$ (uncorrelated). Our aim is to come up with a scheme that will generate $|\psi_M\rangle$ so that the Ramsey sequence can resume with its usual step 2 from this point on. This will give a minimal phase sensitivity of $1/N$.

- We describe a scheme for divalent atoms for creating GHZ states in a Rydberg gas adapted from a scheme described for Rb in Saffman and Mølmer, PRL **102**, 240502 (2009).

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Motivation

How do we generate a *maximally* **correlated** (entangled) state to feed into the 2nd step of the Ramsey sequence?

Distributed quantum network of clocks

PRL 112, 190403 (2014) PHYSICAL REVIEW LETTERS week ending 16 MAY 2014

Heisenberg-Limited Atom Clocks Based on Entangled Qubits

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A quantum network of clocks

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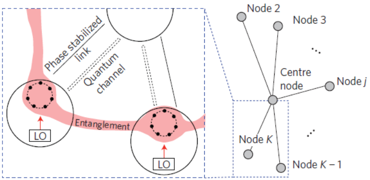
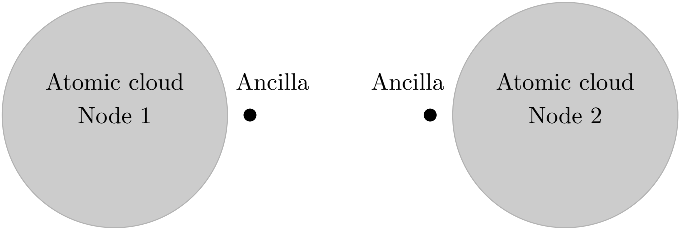


Figure 1 | The concept of world-wide quantum clock network. a, Illustration of a cooperative clock operation protocol in which individual parties (for example, satellite-based atomic clocks from different countries) jointly allocate their respective resources in a global network involving entangled quantum states.

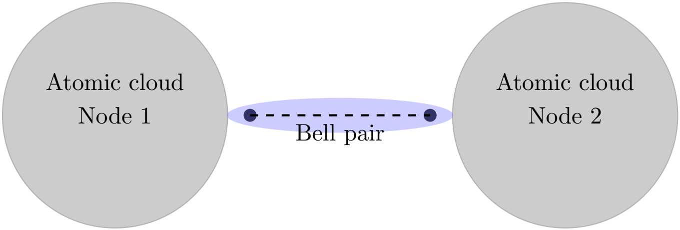
* Create a clock network from a set of spatially separated atomic clocks.
* Preparing these clocks in a single distributed entangled state drastically improves clock stability.
* Can use to establish an international time scale.

Distributed quantum network of clocks (cont'd)



1. The state of the ancilla atom: $|g\rangle_a$.
   The state of the cloud: $|gg\cdots g\rangle_c = |g\rangle_c^{\otimes N}$.

Distributed quantum network of clocks (cont'd)



1. The state of the ancilla atom: $|g\rangle_a$.
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2. Create a Bell pair: $|g\rangle_a \to \frac{1}{\sqrt{2}}(|g\rangle_a + |ns\rangle_a)$
   The system: $|\psi\rangle = |g\rangle_a |g\rangle_c^{\otimes N} + |ns\rangle_a |g\rangle_c^{\otimes N}$
3. Map the ancilla state to the cloud (π-pulse)
   $|\psi\rangle \to |g\rangle_a \left(\sum_j \sigma_j^+ |g\rangle_c^{\otimes N}\right) + |ns\rangle_a |g\rangle_c^{\otimes N}$

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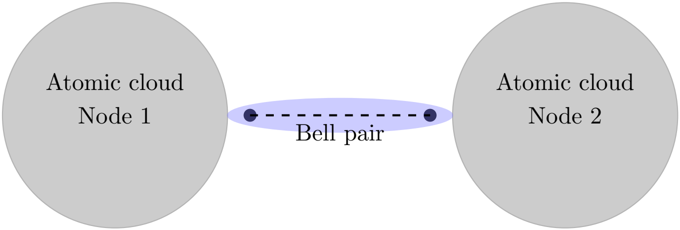

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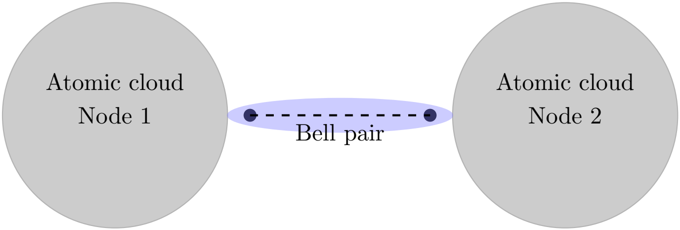

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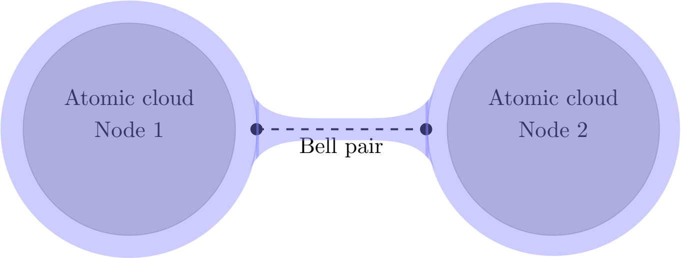

4. We would like to engineer a gate that does $\sum_j \sigma_j^+ |g\rangle_c^{\otimes N} \rightarrow |g\rangle_c^{\otimes N}$ and $|g\rangle_c^{\otimes N} \rightarrow |f\rangle_c^{\otimes N}$.
5. We then end up with the GHZ state:
   $|\psi\rangle \rightarrow |g\rangle_a |g\rangle_c^{\otimes N} + |ns\rangle_a |f\rangle_c^{\otimes N}$.

How do we engineer such a gate?

Entanglement via asymmetric Rydberg blockade

Entanglement via Rydberg blockade - Rb

PRL **102,** 240502 (2009) PHYSICAL REVIEW LETTERS week ending 19 JUNE 2009

Efficient Multiparticle Entanglement via Asymmetric Rydberg Blockade

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(Received 12 December 2008; published 17 June 2009)

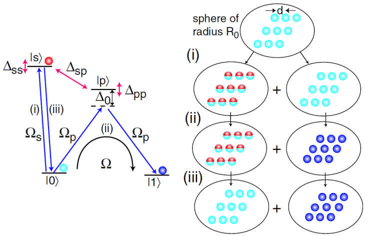
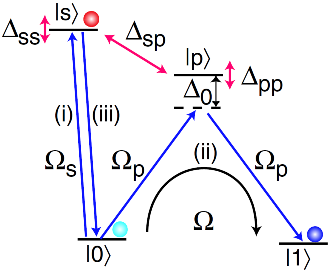


FIG. 1 (color online). Level scheme (left) and sequence of operations for entangled state generation (right). Ω is the effective Rabi frequency coupling states |0⟩, |1⟩.

Conditions:

* $\Delta_{ss} \gg \Omega_s$ (only single particle excitations)
* $\Delta_{pp} \ll \Delta_{ss}$ ($|p\rangle$ interact weakly)
* $\Delta_{sp} \gg \Delta_{pp}$ ($|s\rangle$ and $|p\rangle$ interact strongly) $\Delta_{sp}/\Delta_{pp} > 150$
* $\Delta_{sp} \sim 1/R^3$ $\Delta_{pp} \sim 1/R^6$

Procedure



* Prepare product state $|\psi\rangle = |00,\ldots,0\rangle$
* Rabi frequency $\Omega_s/2 = \langle s|H_1|0\rangle \ll \Delta_{ss}$
* (1) Apply H_1 for $t_1 = \pi/(2\sqrt{N}|\Omega_s|)$

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{N}}\sum_{j=1}^{N}|0,0,s^{(j)},\ldots,0\rangle + |0,0,\ldots,0\rangle\right)$$

* Rabi frequency $\Omega_p/2 = \langle p|H_{20}|0\rangle = \langle p|H_{21}|1\rangle$
* (2) Apply $H_2 = H_{20} + H_{21}$ for $t_2 = 2\pi\Delta_0/\Omega_p^2$ ($\Delta_{pp} \ll \Omega \ll \Delta_{sp}$)

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(\frac{1}{\sqrt{N}}\sum_{j=1}^{N}|0,0,s^{(j)},\ldots,0\rangle + |1,1,\ldots,1\rangle\right)$$

* (3) Apply $-H_1$ for $2t_1$ to undo first step

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|0,0,0,\ldots,0\rangle + |1,1,\ldots,1\rangle\right)$$

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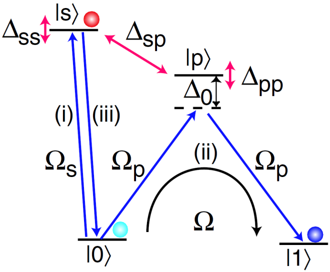

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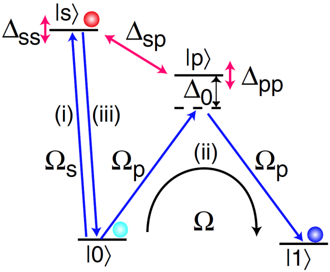

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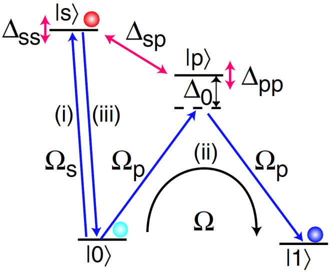

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Side-note: trapping divalent Rydberg atoms in optical lattices

Magic trapping of divalent Rydberg atoms I

Optical potential can be expressed in terms of the **dynamic polarizability**:

$$U(z) = -F_0^2 \alpha(\omega)/4$$

The Ry state dynamic polarizability has a **position (Z) dependent** and a **constant offset** term:

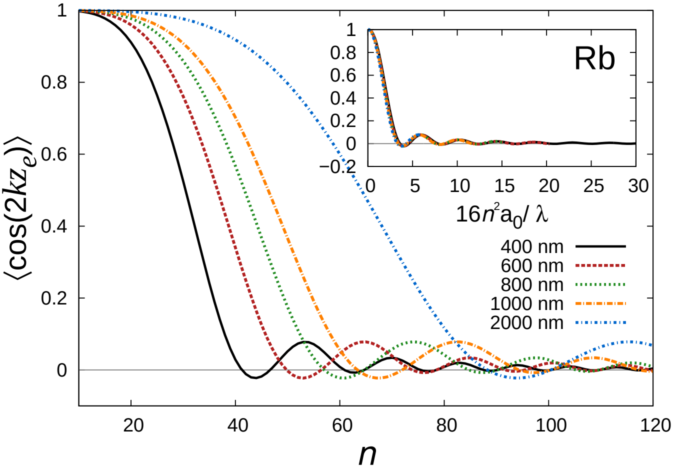
$$\alpha_{GS}(Z,\omega) = \alpha_g(\omega) \sin^2(kZ)$$

$$\alpha_r(Z,\omega) = \underbrace{-\frac{1}{\omega^2}\langle nlm|\cos(2kz_e)|nlm\rangle}_{\alpha_r^{\mathrm{lsc}}(\omega)} \sin^2(kZ) + \langle nlm|\sin^2(kz_e)|nlm\rangle \quad \text{(offset)}$$

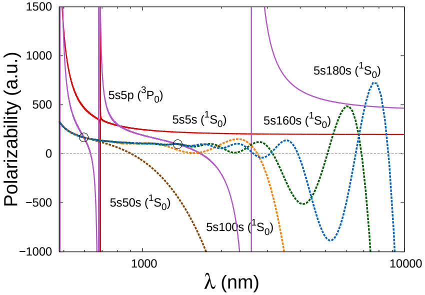
When the Z-dependent contributions to the GS and Ry polarizabilities are matched, the optical potentials seen in the ground state $U_g(z)$ and the Rydberg state $U_r(z)$ only differ by a constant offset.

Magic trapping of divalent Rydberg atoms II

For small n, $\langle\cos(2kz_e)\rangle \to 1$ at a given k, resulting in $\alpha_r^{\rm lsc}(\omega) \to \alpha_e$. **As** n **is increased**, the trapping potential minima switch back and forth between the nodes and the anti-nodes of the lattice.



Magic trapping of divalent Rydberg atoms III



(Figure) $\alpha^{J=0}_{5snl}(\lambda)$ for the 5sns($^1S_0$) (left panel) and 5snp($^3P_0$) (right panel) **Ry states** of Sr for various n-states plotted with the **ground state** 5s^2($^1S_0$) and the **upper clock state** 5s5p($^3P_0$) polarizabilities. Two special points at which the 5s5p($^3P_0$) polarizability matches those of the Ry states in the high-n limit are marked by open circles. These **universal magic wavelengths** are at 596 nm and 1362 nm.

Back to the entanglement scheme

Strontium

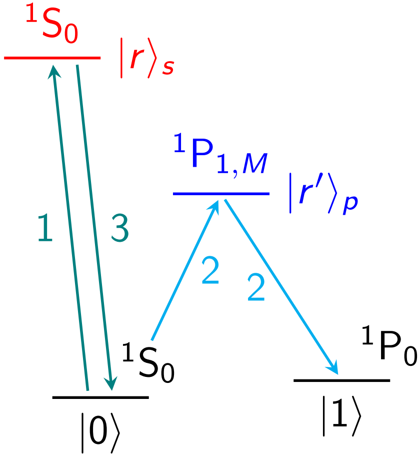


Figure : Generation of the maximally entangled GHZ state in Sr.

Conditions:

* Large $^1S_0 +^1 S_0$ interaction to have large blockade radius. (Van der Waals interaction $\sim C_6/R^6$)
* Strong $|r\rangle$ and $|r'\rangle$ interaction. (Dipole-dipole interaction $\sim C_3/R^3$) $\mathbf{\Delta_{sp} \gg \Delta_{pp}}$
* All atoms not blockaded by $^1S_0 +^1 P_1$ interaction must be able to follow route 2: $\mathbf{\Delta_{pp} \ll \Delta_{ss}}$ (Smaller $C_6(^1P_1 +^1 P_1)$)

The conditions imply relationships between $|r\rangle_s$ and $|r'\rangle_p$

* The condition $\mathbf{\Delta_{ss}} \gg \mathbf{\Delta_{pp}}$ implies:

$$\frac{\widetilde{C}_6^{(ss)} n^{11}}{R^6} \gg \frac{\widetilde{C}_6^{(pp)} n^{11}}{R^6}$$
$$\widetilde{C}_6^{(ss)} \gg \widetilde{C}_6^{(pp)}$$

* The second condition $\mathbf{\Delta_{sp}} \gg \mathbf{\Delta_{pp}}$ provides a range for n:

$$\frac{\widetilde{C}_3^{(sp)} n^4}{R^3} \gg \frac{\widetilde{C}_6^{(pp)} n^{11}}{R^6} \quad \rightarrow \quad n \ll \left(\frac{\widetilde{C}_3^{(sp)}}{\widetilde{C}_6^{(pp)}} R^3 \right)^{1/7}$$

R **is the inter-atomic distance set by the experimental configuration.**

Practical set-up in a one-dimensional optical lattice

Experimental configuration

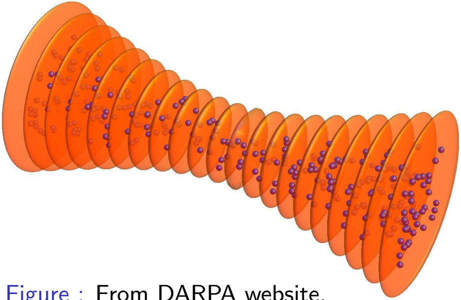


Figure : From DARPA website.
See Science **341**, 6151 (2013)

* 1D trapping in an optical lattice
* Spatial extent of harmonic trap $\sqrt{\hbar/m\omega}$ with $N = 20$ atoms per site
* $f_z = 80$ KHz $\rightarrow \Delta z = 27$ nm
* $f_r = 450$ Hz $\rightarrow \Delta r = 504$ nm
  [Jun Ye Group, Nature, DOI 10.1038/nature12941]

* This gives a **volume per atom** inside a "pancake": $V = \dfrac{\pi(\Delta r)^2 \Delta z}{N}$, and a mean inter-atomic distance $d \simeq (6V/\pi)^{1/3} = 127$ nm.
* Loosening the trap increases d: *e.g.*
  $f_z = 40$ KHz and $f_r = 225$ Hz yields $d \simeq 180$ nm.

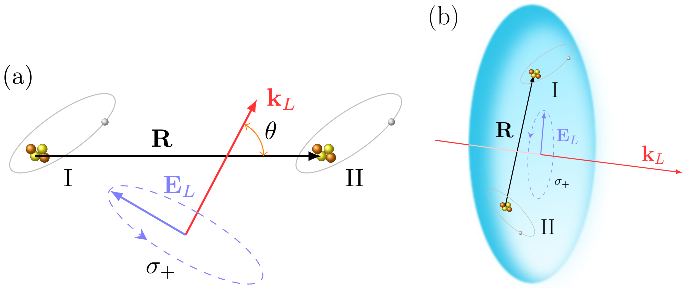
Experimental configuration - size considerations

* These size scales alone impose a constraint on n: $2n^2 < d/2$.
  - $f_z = 80$ KHz and $f_r = 450$ Hz trap allows for $n < 25$.
  - $f_z = 40$ KHz and $f_r = 225$ Hz trap allows for $n < 30$.
  - $f_z = 20$ KHz and $f_r = 112$ Hz trap allows for $n < 35$.

* One possibility of increasing the inter-atomic distances is by going to 3D optical lattice geometry. Leaving empty lattices between trapped single atoms would allow us to adjust the inter-atomic separation.

* The main issue here is the high density of atoms restricting n too low. To reduce the density in the cloud, we can try a **MOT** or an **optical dipole trap** rather than an optical lattice.

Diagram



1. (a) A σ_+-polarized excitation laser whose wavevector $\mathbf{k}_L$ is at an angle θ with the interatomic axis $\mathbf{R}$ is driving the transition from $5s^2(^1S_0)$ ground state to the $5snp(^1P_{1,M_{k_L}=1})$ Rydberg state with $\Delta M_{k_L} = +1$. Here M_{k_L} is the projection of the total angular momentum onto the axis defined by $\mathbf{k}_L$, which makes $\mathbf{k}_L$ the quantization axis.
2. (b) The atoms are inside a pancake shaped atomic cloud in a 1D optical lattice: $\theta = \pi/2$ and $\mathbf{k}_L$ is parallel to the wave vector $\mathbf{k}$ (along the x-axis).

General scheme

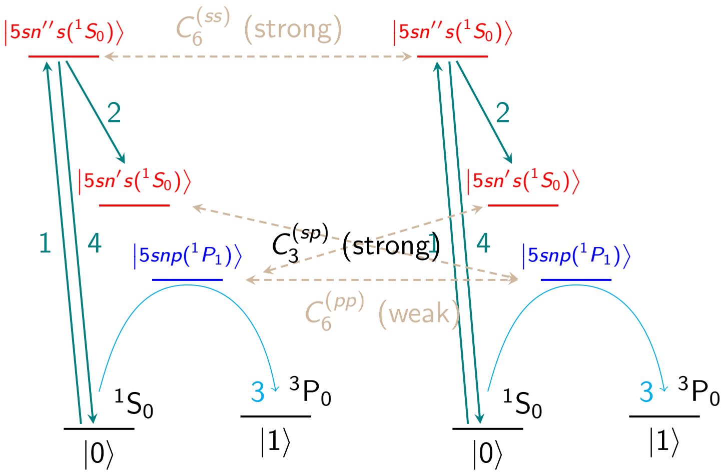


Figure : Generation of the maximally entangled GHZ state in Sr.

Choose states

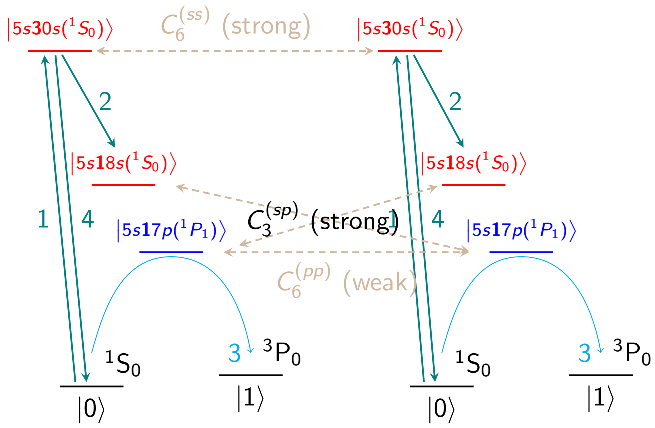


Figure : Choose n'' as large high as possible as permitted by the atomic density of the cloud ($n'' = 30$). Then choose $5snp(^1P_1)$ such that it has the smallest possible $\widetilde{C}_6$ ($n = 17$). Having chosen $5snp(^1P_1)$, choose $5sn's(^1S_0)$ so that it has the largest $\widetilde{C}_3$ (*i.e.* the largest overlap) with the $^1P_1$ state ($n' = 18$).

Interaction strengths

* $5s30s(^1S_0) + 5s30s(^1S_0)$: $\Delta_{ss}^{(30)} = -\frac{9.4}{R^6} n^{11}$
* $5s17p(^1P_{1,1_x}) + 5s17p(^1P_{1,1_x})$: $\Delta_{pp} = -\frac{0.24}{R^6} n^{11}$
* $5s18s(^1S_0) + 5s17p(^1P_{1,1_x})$: $\Delta_{sp} = \frac{0.09}{R^3} n^4$

Following are the asymmetry conditions between the interaction strengths where the interatomic separation $R = d$

* $\Delta_{sp} \gg \Delta_{pp}$: $\Delta_{sp}/\Delta_{pp} \simeq 1.1 \times 10^{-9}\ d^3$
  - $f_z = 40$ KHz and $f_r = 225$ Hz gives $d = 180$ nm: $\Delta_{sp}/\Delta_{pp} \simeq 45$
  - $f_z = 20$ KHz and $f_r = 112$ Hz gives $d = 254$ nm and $\Delta_{sp}/\Delta_{pp} \simeq 127$.
* Also, we need to have Δ_{ss} and Δ_{sp} to each define a blockade radius that is at least as large as the size of the atom cloud. The bloackade raidus is defined by Δ_{ss} and Δ_{sp} and the Ω of the excitation lasers.

Long-range interactions between the Rydberg atoms

$\widetilde{C}_6$ coefficients for ${}^1S_0 + {}^1S_0$ and ${}^1P_1 + {}^1P_1$ interactions

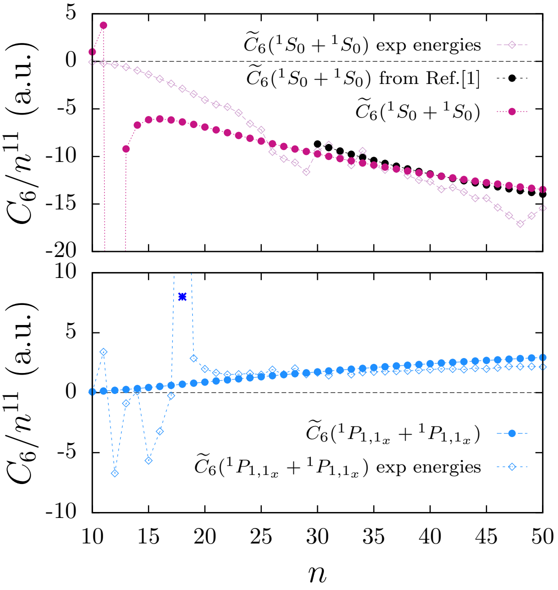


Figure : $\widetilde{C}_6$ coefficients for the ${}^1S_0 + {}^1S_0$ and ${}^1P_1 + {}^1P_1$ interactions in Sr as a function of n. The solid and empty points differ in the energy denominators used to calculate them: the solid points use the numerical energies obtained from a model potential whereas the empty points use experimentally available values. Negative C_6 imply attractive interactions. Black points in top panel are from **JPB 45, 135004 (2012)**. The blue star labels $\widetilde{C}_6 \simeq 37$ a.u. for $n = 18$.

The asymmetry condition $\boldsymbol{\Delta_{sp} \gg \Delta_{pp}}$

1. The condition $\boldsymbol{\Delta_{sp} \gg \Delta_{pp}}$ implies:

$$\frac{\widetilde{C}_3^{(sp)} n^4}{R^3} \gg \frac{\widetilde{C}_6^{(pp)} n^{11}}{R^6} \quad \rightarrow \quad n \ll \left(\frac{\widetilde{C}_3^{(sp)}}{\widetilde{C}_6^{(pp)}} \right)^{1/7} R^{3/7} \equiv n_{\mathrm{max}}$$

   This shows how high up in n can we go and still abide by the second condition $\Delta_{sp} \gg \Delta_{pp}$. Note that this is on top of the condition set by the mean interatomic distance.

2. The ratio of the scaled C_N coefficients $\left[\widetilde{C}_3^{(sp)} / \widetilde{C}_6^{(pp)}\right]$ has *residual* dependence on n because the atom is not hydrogen. This makes the upper bound for n suggested by the condition $\boldsymbol{\Delta_{sp} \gg \Delta_{pp}}$ depend on n itself. However, this dependence is not strong because the upper limit on n is set by the 7th root of this ratio and the ratio itself has weak residual dependence on n.

Residual n-dependence of $\left[\widetilde{C}_3^{(sp)}/\widetilde{C}_6^{(pp)}\right]$ and $n_{\max}$

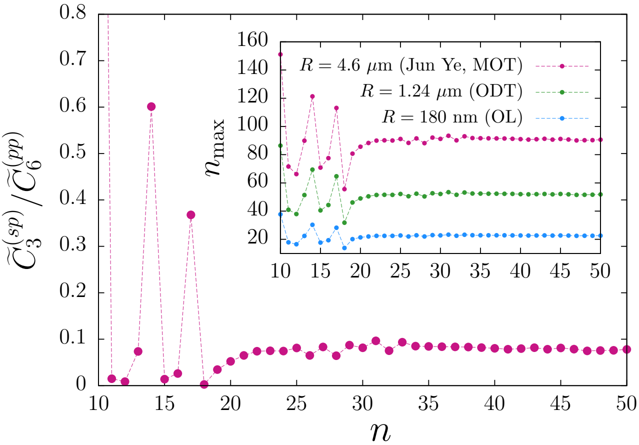


Figure : Residual n-dependence of the ratio $\widetilde{C}_3^{(sp)}/\widetilde{C}_6^{(pp)}$ (violet). The inset shows the upper limit for n imposed by the asymmetry condition $\Delta_{sp} \gg \Delta_{pp}$.

Quadrupole-Quadrupole interaction for $^1P_1 +^1 P_1$

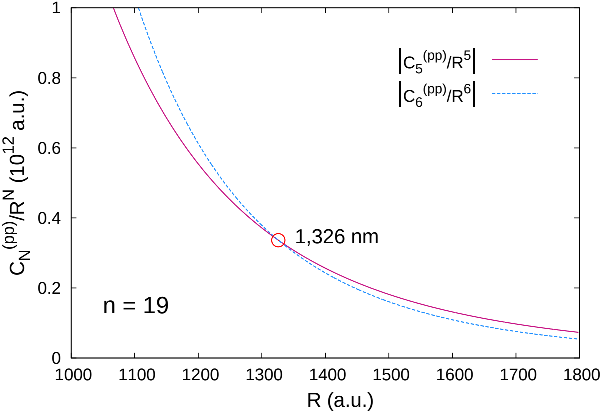


Figure : $R = 179$ nm $\simeq 3390$ a.u.. For the experimental distance scales, the quadrupole-quadrupole interaction is small compared to the van der Walls interaction between the $^1P_1$ states. This allows us to ignore the quadrupole term in the first condition: $\widetilde{C}_6^{(ss)} n^{11}/R^6 \gg \widetilde{C}_6^{(pp)} n^{11}/R^6 + \widetilde{C}_5^{(pp)} n^8/R^5$ because $[\widetilde{C}_5^{(pp)} n^8/R^5]/[\widetilde{C}_6^{(pp)} n^{11}/R^6] \simeq 0.14$ for $n = 19$ at $R = 179$ nm.

Quadrupole-Quadrupole interaction for ${}^1P_1 + {}^1P_1$

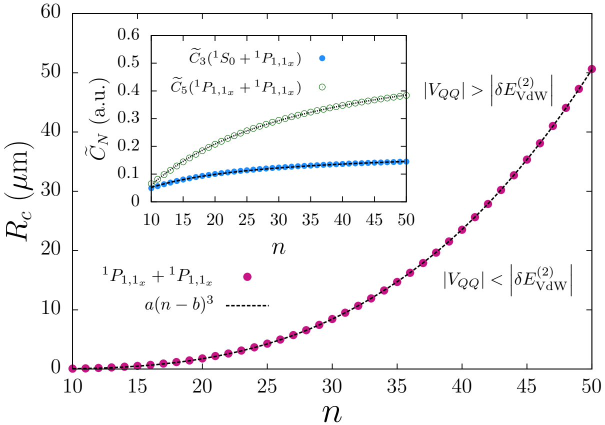
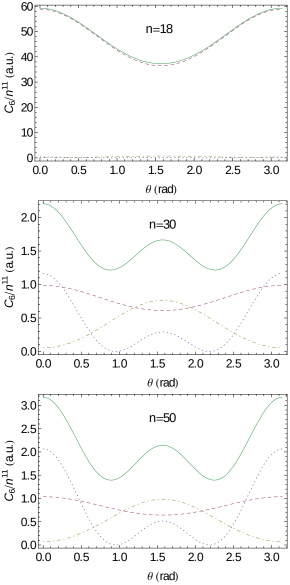


Figure : The point here is that neglecting the quadrupole-quadrupole interaction between the ${}^1P_1$ states is well justified for the length scales set by the experimental setup. One has to place the atoms over a 1000 nm apart for the quadrupole-quadrupole and the van der Waals interactions to have the same strength.

Angular dependence of the long-range interactions



Dipole-dipole interactions: $V_{DD} = C_3/R^3$

$$C_3 = \frac{1}{3} P_2(\cos\theta) |\langle {}^1S_0 || D || {}^1P_1 \rangle|^2$$

Quadrupole-quadrupole interactions:
$V_{QQ} = C_5/R^5$

$$C_5 = \frac{1}{5} P_4(\cos\theta) |\langle {}^1P_1 || D || {}^1P_1 \rangle|^2$$

van der Waals interactions: $V_{vdW} = C_6/R^6$

$$c_1 + c_2 P_2(\cos\theta) + c_3 P_4(\cos\theta)$$

- $^1P_1 + {}^1P_1 \rightarrow {}^1S_0 + {}^1S_0$ channel
- $^1P_1 + {}^1P_1 \rightarrow {}^1D_2 + {}^1D_2$ channel
- $^1P_1 + {}^1P_1 \rightarrow {}^1S_0 + {}^1D_2$ channel

Exploiting the series perturbation in divalent atoms

$\Delta_{sp} \gg \Delta_{pp}$: Exploiting series perturbation in Sr

The condition $\Delta_{sp} \gg \Delta_{pp}$ implies

$$\frac{\Delta_{sp}}{\Delta_{pp}} = \left(\frac{\widetilde{C}_3^{sp}}{\widetilde{C}_6^{pp}} \right) \frac{d^3}{n^7} \gg 1 \, .$$

If the usual n-scaling were to be true for all n of interest, we would want to choose d as large as possible and n as low as possible to obtain the largest ratio Δ_{sp}/Δ_{pp}. **For $n < 20$ in Sr however, this scaling breaks down and we are presented with new opportunities for even larger ratio Δ_{sp}/Δ_{pp} than otherwise possible with the hydrogenic n-scaling.**

This is why we picked $n = 17$ for the $5snp(^1P_{1,1_x})$ state earlier, because the $\widetilde{C}_6^{(pp)}$ coefficient for this state is abnormally small due to the highly perturbed nature of the $5snd(^1D_2)$ intermediate states in the $n < 20$ region.

Next slide shows a plot of the ratio Δ_{sp}/Δ_{pp} as a function of R and n in the n-range of interest.

Series perturbation in the $^1P_1 + {}^1P_1$ interactions

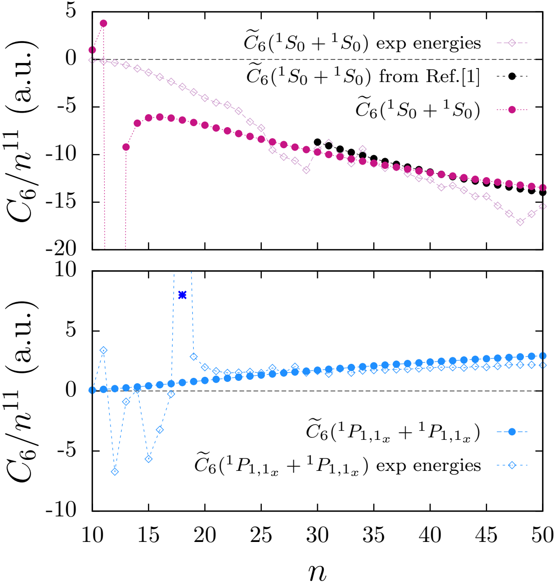


Figure : $^1P_1 + {}^1P_1$ van der Waals interaction in Sr has intermediate states $^1S_0 + {}^1S_0$, $^1D_2 + {}^1D_2$ and $^1S_0 + {}^1D_2$. The $5snd(^1D_2)$ series is pertubed by the $4d6s(^1D_2)$ between $n = 11 - 17$. The $4d^2(^1D_2)$ also pertubs $5s12d(^1D_2)$.

The ratio Δ_{sp}/Δ_{pp}

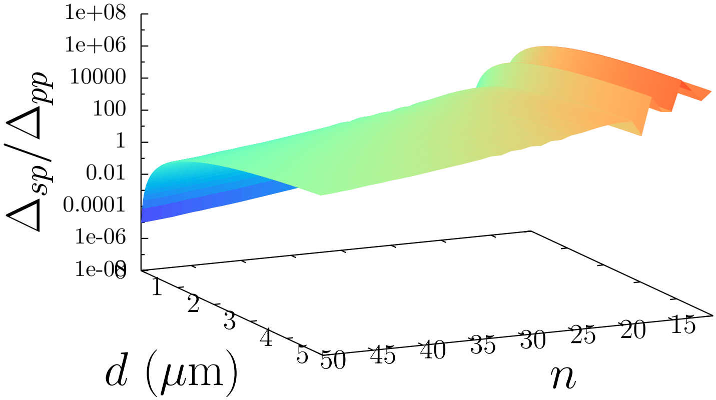
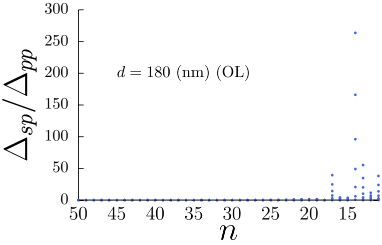
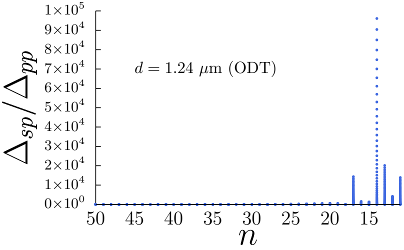
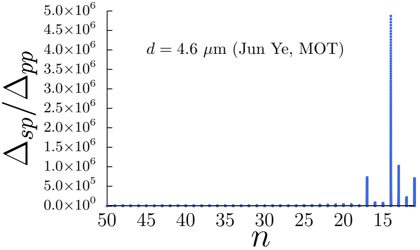


Figure : The ratio Δ_{sp}/Δ_{pp} as a function of R and n. The next slide shows constant R cuts in this surface, which correspond to typical interatomic distances in optical lattices, MOTs and ODTs.

The ratio Δ_{sp}/Δ_{pp} at specific d







“Other” considerations ...

Dipole-Dipole interaction vs Radiative lifetime

* Energy shift due to dipole-dipole interaction ($n = 20$)

$$\left| \frac{\widetilde{C}_3^{(sp)}}{R^3} \right| = \frac{0.18}{R^3} n^4 \simeq 78 \;\; \text{GHz}$$

where $R = 179$ nm.

* Lifetime for the $5s15s(^1S_0)$ state of Sr is 745 ns.
This results in a natural line broadening of 10^{-4} GHz, which well resolves the dipole-dipole shift.

* Lifetime for the $5s19p(^1P_1)$ state of Sr is 1,890 ns.
This results in a natural line broadening of 4×10^{-5} GHz.

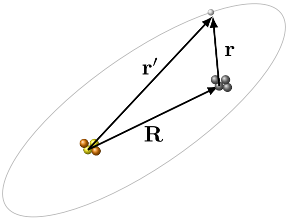
* Natural broadening also sets a limit for the blockade radius. For the $5s15s(^1S_0)$ state:

$$R_{blockade} = \left(2\tau \widetilde{C}_3^{(sp)} \right)^{1/3} \simeq 4.3 \; \mu\text{m} \; .$$

Short-range interactions I

* Overall energy shift for all atoms due to isotropy. Only important at the edges.
* We would like to evaluate the energy shift experienced by a Rydberg atom due to a ground state atom in its immediate vicinity. The ground state atom is assumed to be embedded in the Rydberg electron cloud.

$$\Delta E = \int \psi^*(\mathbf{r}')V(\mathbf{r})\psi(\mathbf{r}')d^3\mathbf{r}'$$



The ground state atom is positioned at $\mathbf{R}$ with respect to the center of the Rydberg atom and the coordinate of the Rydberg electron is $\mathbf{r}'$. The position vector of the electron in the frame of the ground state atom is $\mathbf{r}$.

$$\Delta E = \int \psi^*(\mathbf{R}+\mathbf{r})V(\mathbf{r})\psi(\mathbf{R}+\mathbf{r})d^3\mathbf{r}$$

Short-range interactions II

* ΔE sits on top of a constant background of the polarization potential from the Sr atom, $-\alpha_{\mathrm{Sr}}/(2r^4) \approx 200$ Hz.
* An electron scattering off of an atom can be described using the Fermi pseudo-potential:

$$V(\mathbf{r})\psi_{ns}(\mathbf{R}+\mathbf{r}) \simeq \frac{2\pi\hbar^2 a_s}{\mu}\delta(\mathbf{r})\psi_{ns}(\mathbf{R}) \, ,$$

which gives

$$\Delta E = \frac{\hbar^2 a_s}{2\mu R^2}|P_{ns}(R)|^2 \, .$$

* To compare these with the shifts due to dipole-dipole interactions, $|C_3^{(sp)}/R^3| \sim 0.65$ GHz between $5s40s(^1S_0)$ and $5s39p(^1P_1)$ states of Sr when the atoms are separated by 1 μm (the radius of $n = 40$ state is $\sim$0.17 μm).

Short-range interaction potential

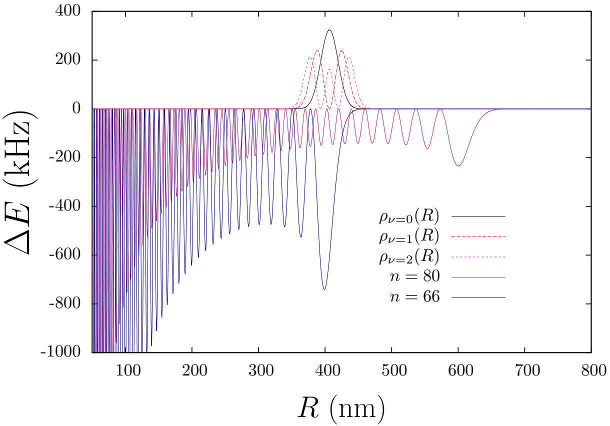


Figure : ΔE for various $5sns(^1S_0)$ states of Sr. The horizontal dashed line marks 300 kHz. The spatial extent of the motional ground state is $d \approx \sqrt{\langle z^2 \rangle} \simeq 13$ nm.

Short-range interactions between trapped atoms

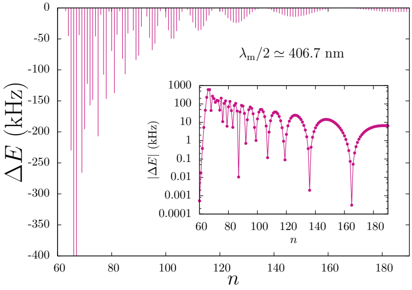


Figure : ΔE experienced by a Rydberg atom separated from a ground state atom by 813.4 nm in its neighboring lattice site as a function of n. The red impulses are calculated using ΔE and are in the kHz regime. The inset shows the absolute values of ΔE from the larger panel plotted in logarithmic scale to clearly read-off sizes of the shifts.

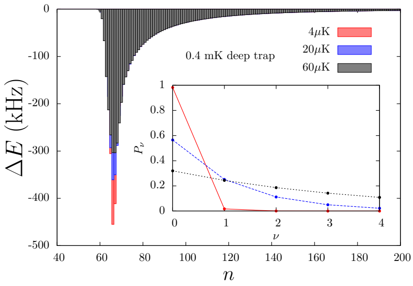


Figure : Energy shift after thermal averaging between the low lying motional states in a 0.4 mK deep optical trap. Inset shows the distribution of the populations between the motional states in the trap.

Summary and outlook

* Fidelity of the entangling gate
* Creating Bell pairs over long distances (DLCZ scheme)
* Effect of ionization due to lattice lasers on the fidelity
* Repeat for Yb: appealing due to existence of telecom wavelength transitions ($\lambda_T \simeq 1.5\ \mu$m)